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, Alain JEAN-MARIE
Fabien PRIEUR
Mabel TIDBALL

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Are Pollution Permits a Cure for Unregulated Growth Diseases?¹

Alain Jean-Marie², Fabien Prieur³ and Mabel Tidball⁴

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²INRIA and LIRMM, 161 rue Ada 34392 Montpellier Cedex 5 - France. E-mail: ajm@lirmm.fr

³University of Savoie, INRA-LAMETA and GREQAM, 2, rue de la charité 13206 Marseille, France. Phone number: 0033499012723 and e-mail: prieur@supagro.inra.fr.

⁴INRA-LAMETA, 2, place Viala 34000 Montpellier, FRANCE. E-mail: tidball@supagro.inra.fr.
Abstract.- We consider an OLG model with emissions arising from production and potential irreversible pollution. Pollution control goes through a system of permits and private agents can also maintain the environment. In this setting, we prove that there exist multiple equilibria. Due to the possible irreversibility of pollution, the economy can be dragged into both environmental and poverty traps. First, we show that choosing a global quota on emissions at the lowest level beyond a critical threshold is a means to avoid the two types of traps. Next, we analyze the impact of a political reform on other equilibria. When the agents do not engage in maintenance, a fall in the quota implies a reduction of pollution but is detrimental to capital accumulation while, in the other case, it procures a double dividend.

Key words: overlapping generations, irreversible pollution, poverty trap, environmental trap, pollution permits

JEL codes: D91, D62, Q28, Q56, O11.
1 Introduction

The signature of the Kyoto Protocol (1997) for the reduction of greenhouse gas emissions is a sign of the interest and the trust placed in the deployment of a pollution permits system. From a theoretical point of view, the guidelines for guaranteeing the functioning of this instrument are still a matter of debate. As far as the control of polluting emissions is concerned, the efficiency of this regulation model is not disputable. Indeed, since the work of Montgomery [1972] or Baumol and Oates [1988], we know that it is sufficient for the regulator, to choose a global emission quota (in function of some predefined objective) and let the market act, in order to guarantee an optimal allocation of permits between polluting firms. But among other discussion topics, the question of the impact of a regulation via permits on the economic development process remains, to a large extent, open.

There exists a large body of literature, dedicated to the analysis of the impact of a reform of the environmental policy on economic growth, based on the assumption that agents have an infinite lifespan. However, these papers concentrate exclusively on the tax instrument (see in particular Bovenberg and Smulders [1993], [1996] or Bovenberg and de Mooij [1997]). A feature common to all these papers is the introduction of an environmental externality in production, which translates the idea that the quality of the environment should improve the productivity of private inputs. Their essential conclusion is that a more ambitious policy (that is, a raise in the tax on polluting emissions) may provide a double dividend: a simultaneous increase of environmental quality and the growth rate, provided that the environment has a strong positive impact on the technology.

On the contrary, few studies consider this problem for a regulation through permits, with the notable exceptions of Jouvet, Michel and Vidal [2002] and Ono [2002]. These papers also differ from the preceding ones by the formalism adopted. Indeed, their objective is to measure the macroeconomic consequences of a strengthening of the permit system (that is, a decrease in the global emission quota) in the setting of an overlapping generations model, by considering the pollution as a production factor. In Jouvet, Michel and Vidal [2002], the consequence of a more severe policy depends essentially on technological parameters. A decrease in the quota penalises (respectively, stimulates) the accumulation of capital when the inputs are complements (respectively, strong substitutes). For moderate values of the elasticity of substitution, the direction of the global impact remains undetermined. Ono [2002] shows that a decrease in the emission quota allocated to polluters may even have, in the long run, an effect contrary to what is expected, by provoking a decrease in the capital level and an increase in the pollution level. Our contribution extends this study to the situation where there is a risk of irreversible pollution.

Indeed, we choose to assess this issue through an extension of the model of Prieur [2007]. This is an OLG model where consumption is the source of emissions. Pollution stock accumulates with emissions but accumulation can be partly controlled by environmental maintenance. In addition,
this model breaks with the standard approach, which is systematically followed by related papers such as John and Pecchenino [1994], assuming a constant-rate assimilation of pollution by Nature. Instead, inspired in particular by Foster [1975] or Tahvonen and Wihagen [1996], it is assumed that the waste assimilation capacity is inverted U-shaped and vanishes beyond a given threshold of pollution. By considering the potential irreversibility of environmental damages, our aim was to propose an original analysis of the relationship between growth and the environment. One of the main results was that a non-regulated growth process may lead a polluting economy into a poverty trap, both economic and ecological, despite a pollution abatement activity operates.

The existence of such a long-term state confirms the necessity for an intervention of the Public Authorities for the management of pollution problems, and leads us to study more deeply the means and the consequences of such an intervention. For this purpose, we develop further the previous model, assuming that the pollution proceeds from the production activity, and that it is controlled thanks to a pollution permit market. In this context, the question is first to know under which conditions this system of regulation allows to avoid the drift of an economy towards a poverty trap. Next, we seek to assess the effects of a reform of the pollution permit system on the growth perspectives of an economy.

The equilibrium analysis stresses on the existence of multiple equilibria of different nature. The first type of equilibrium has the features of environmental and/or poverty traps. To be more precise, a distinction is made between stationary traps, that correspond to steady states with irreversible pollution and/or low wealth, and asymptotic traps that refer to equilibrium trajectories accompanied by a continuous erosion in economic and/or environmental resources. Our purpose, in this paper, is first to address the efficiency of the permits system in terms of its ability to prevent the economy from reaching these undesirable outcomes. Actually, we prove that fixing the global quota on emissions above a critical level succeeds in avoiding stationary traps. In other words, the preliminary recommendation is to allow firms to pollute sufficiently. It does not mean however that firms can pollute as much as they want. In fact, if one excludes the hopeless situation where the environment is irreversibly degraded when the permit system is set up, then further investigations reveal that the quota has to be chosen at the lowest level of the critical threshold in order to protect the economy against asymptotic traps.

Having answered our first question of interest, the emphasis is next put on the analysis of the second class of equilibria. It comprises steady states that are called desirable in the sense that they are relatively wealthy and exhibit reversible pollution. Our aim is to evaluate the impact of a political reform on their properties. It turns out that the results depend on whether agents engage in maintenance, or not. As for zero maintenance equilibria (ZME), there exists a compromise between capital accumulation and pollution control. If lowering the emissions quota indeed succeeds in reducing pollution, it also generates a negative effect on capital accumulation. Concerning positive maintenance equilibria (PME), the key element is the evolution of the balance between financial and environmental constraints imposed to the agents. In fact, at the locally
stable equilibrium, we show that the economy enjoys a double dividend: a reduction of the quota allows the economy to reach a long-term state that is both richer and less polluted. And, the derivation of such a result does not necessitate to resort to the controversial assumption of a positive environmental externality on production, as opposed to the literature on pollution tax reforms (Bovenberg and Smulders [1995], [1996]...).

In summary, aside from the compromise associated with ZME, the general recommendation that can be deduced from our analysis is to set the global emission quota as small as possible above a certain threshold, thereby excluding poverty traps and enhancing the attributes, in term of wealth and pollution, of desirable PME.

The paper is organized as follows. Section 2 sets out the model. Section 3 derives the equilibrium and analyzes the impact of a political reform on the equilibrium properties. Section 4 performs some numerical simulations so as to outline the implications of a change in policy on the global dynamics and, notably, on the possibility to reach a safe and wealthy steady state. Finally, Section 5 concludes.

2 The model

We develop an overlapping generations model à la Allais [1947], Samuelson [1958] and Diamond [1965]. In a perfectly competitive world, firms produce a single homogeneous good used both for consumption and investment. The production process generates harmful polluting emissions. Pollution control goes through the implementation of a policy consisting in both the definition of a global emission quota at each period $\bar{E}_t$ and the creation of an exchange market for pollution permits. More precisely, we assume that the quota imposes upon the economy in an exogenous manner. The level of emission to be respected, for instance, is decided during international negotiations (like the Kyoto protocol (1997)) where all participants promise to reduce their emissions. The government’s role is thus limited to sell a volume of pollution permits corresponding to $\bar{E}_t$ to the polluting firms. It is also responsible for the distribution of the income obtained from the sale of permits (the environmental allowance) to households. In addition, following Ono [2002], we assume that the households can also engage in environmental maintenance.

2.1 Pollution dynamics

In the absence of human activity, pollution accumulation, for non-negative levels of the stock $P_t$, is described by the following equation:

$$P_{t+1} = P_t - \Gamma(P_t)$$

(1)

where $\Gamma(P_t)$ corresponds to the natural decay function that gives the amount of pollution assimilated by nature each period. Nature’s ability to absorb pollution depends on the level of pollutant
concentration. More precisely, our aim is to express the idea that too high levels of pollution alter the environment’s recovery process in an irreversible way. Therefore, following Forster [1975], Cesar and de Zeeuw [1994] and Talvonen and Withagen [1996], we assume an inverted U-shape decay function (see fig.1) whose properties, summarized in the assumption below, give an account of the potential irreversibility of environmental damages caused by pollution:

**Assumption 1.** The decay function $\Gamma(P) : \mathbb{R}^+ \rightarrow \mathbb{R}^+$ is $C^2$ over the interval $[0, \bar{P}]$. It is first increasing from $\Gamma(0) = 0$ to a level $\Gamma_{\text{max}} = \Gamma(\bar{P})$ then, decreasing until the pollution reaches the irreversibility threshold $\bar{P}$ ($\Gamma'(P) \geq 0 \forall P \in [0, \bar{P})$, $\Gamma'(P) < 0 \forall P \in (\bar{P}, \bar{P})$ with $\bar{P} < \bar{P}$). Beyond this value, assimilation is nil: $\Gamma(P) = 0 \forall P \geq \bar{P}$. We also assume that $\Gamma'(P) < 1$, which will imply that $\Gamma(P) < P$ for all $P > 0$, that is: the amount of pollution assimilated at each period is lower than the stock.

For low pollution levels, the volume of pollution absorbed by nature is first growing with the stock until reaching a maximal absorption rate $\Gamma_{\text{max}}$ at $P = \bar{P}$. Then, beyond the turning point $\bar{P}$, the regeneration capacity starts to decline and assimilation decreases with the pollutant concentration. Finally, as soon as pollution reaches the critical level $\bar{P}$, the natural rate of decay is nil and pollution accumulation becomes irreversible. In other words, once the stock of pollution has achieved the critical threshold $\bar{P}$, the recovery process of nature is completely and permanently overwhelmed.

We now turn to the analysis of the private agents’ choices and trade-off.

### 2.2 Production

Under perfect competition, the firms produce the final good $Y_t$ with a constant returns to scale technology using labor $L_t$ and capital $K_t$:

$$Y_t = \tilde{\eta}z_t K_t^{\nu} L_t^{1-\nu}$$

(2)
where \( z_t \in [0,1) \) is an index of the technology’s intensity of pollution, \( \tilde{A} > 0 \) corresponds to a productivity scalar and \( \nu \in (0,1) \) is a constant parameter.

Production activity leads to polluting emissions. Following Stokey [1998], the flow of emissions writes:

\[
E_t = z_t^{\nu} Y_t
\]  

with \( \zeta > 0 \). Therefore, the emissions-output ratio is positively linked to the index \( z_t \).

Finally, a combination of equations (2) and (3) allows us to express the production function as constant returns technology with respect to capital, labor and emissions:

\[
Y_t = AK_t^\alpha L_t^\beta E_t^{1-\alpha-\beta}
\]  

with \( A = \tilde{A}^{\nu} \), \( \alpha = \frac{\nu \zeta}{1+\xi} \), \( \beta = \frac{(1-\nu)\zeta}{1+\xi} \).

In Jouvet, Michel and Rotillon [2005], the authors highlight, in a welfare maximization perspective, the superiority of a system of auctions over the principle of a free allocation of permits to firms by showing that the latter is a source of economic distortions. On the basis of their results, we assume that the total amount of permits is auctioned. Note that Ono [2002] considers, on the contrary, that a part of the quota is allocated freely to firms that can next participate to the permits market transactions. However his approach is in fine rigorously identical to ours since, in his model’s equilibrium, remains an income \( q_t \bar{E}_t \), exactly equal to the revenue coming from the sale of the whole quota, which is entirely taxed and paid back to the young households. Thus, firms are obliged to purchase, at the market price \( q_t \), the amount of pollution permits that corresponds precisely to their own need \( E_t \) in order to be able to produce.

We assume that capital fully depreciates in one period. Firms maximise profits, taking the price of inputs as given:

\[
\pi_t = AK_t^\alpha L_t^\beta E_t^{1-\alpha-\beta} - w_t L_t - R_t K_t - q_t E_t
\]  

where \( w_t \) represents the wage rate, \( R_t \) is the interest factor and \( q_t \) is the price of permits.

The first order conditions for profit maximization, expressed in terms of per capita variables with \( k_t = K_t/L_t \) and \( e_t = E_t/L_t \), write:

\[
w_t = \beta A k_t^{\alpha} e_t^{1-\alpha-\beta}
\]

\[
R_t = \alpha A k_t^{\alpha-1} e_t^{1-\alpha-\beta}
\]

\[
q_t = (1 - \alpha - \beta) A k_t^{\alpha} e_t^{-\alpha-\beta}.
\]

The next subsection is devoted to the analysis of the households’s choices et trade-offs.
2.3 The households

We consider an infinite horizon economy composed of finite-lived agents. A new generation is born at each period \( t = 1, 2, \ldots \), and lives for two periods: youth and old age. There is no population growth and the size of a generation is normalized to one \( N = 1 \). The young agent born at period \( t \) is endowed with one unit of labor that she inelastically supplies to firms for a real wage \( w_t \). Her first period income is also composed of the revenue from the sale of a quantity \( E_t \) of permits, at the price \( q_t \). This revenue corresponds to the environmental allowance distributed by the government. She divides this total income between savings \( s_t \) and environmental maintenance \( m_t \).\(^1\) When retired, the agent supplies her savings to firms and earns the return of savings \( R_{t+1}s_t \). Her income is entirely devoted to consumption \( c_{t+1} \).\(^2\) The two budget constraints she faces, in both periods of life, write respectively:

\[
\begin{align*}
  w_t + q_t E_t &= s_t + m_t \\
  c_{t+1} &= R_{t+1}s_t.
\end{align*}
\]

Following Ono [2002], we assume that the pollution abatement activity, driven by households, remains effective despite the permit system. Therefore, the economy has two distinct means for fighting against pollution. If the environmental policy is above all intended to regulate emissions, it also affects, through the distribution of the environmental allowance, the households’ maintenance effort. One may note that this additional income also stimulates productive investment, through savings.

The preferences of the agent born at date \( t \) are defined over old age consumption and environmental quality. They are described by the following utility function \( U(c_{t+1}, P_{t+1}) \):

**Assumption 2.** The utility function \( U(c, P) : \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R} \) is \( C^2 \) with: \( U_1 \geq 0, \quad U_2 \leq 0, \quad U_{11}, U_{22} \leq 0. \) The cross derivative is negative \( U_{12} \leq 0. \)\(^3\) We further assume that \( \lim_{c \to 0} U_1(c, P) = + \infty \).

Emissions contribute to the accumulation of the pollutant stock. It is also possible to control the periodic flux of emissions and to improve environmental quality through the abatement expenditures of the young \( m_t \). Real emissions are simply represented by the following linear function: \( \Theta_t = E_t - \gamma m_t \) with \( 0 \leq \gamma < 1 \). In the presence of human activity, the evolution of pollution (1) then becomes:

\[
P_{t+1} = P_t - \Gamma(P_t) + E_t - \gamma m_t. \tag{11}
\]

\(^1\)It is possible to reinterpret \( m_t \) as a tax levied by a one period lived government in order to finance the abatement activity, for the benefit of agents living during its period of office (John et al. [1993]).

\(^2\)We do not consider any first period consumption. This simplifying assumption allows us to focus on the crucial trade-off between final good and environmental good consumptions (see next page the representative agent problem). Anyway, adding a first period consumption would not change our qualitative results.

\(^3\)Pollution exerts a "distaste" effect on consumption (Michel and Rotillon [1993]). The marginal utility of consumption is decreasing in \( P \) which means that the higher the pollution, the lesser the consumption.
In this framework, households typically face an intergenerational externality. When the young agent chooses the amount of resources to devote to maintenance, she only cares about the environment she will enjoy in old age. But, the agent ignores future benefits of her investment.

The representative agent born at date \( t \) shares her first period income among savings (which determines the consumption of the final good) and maintenance (which influences the "consumption" of the environmental public good) in order to maximise her lifetime utility. Taking as given prices and pollution at the beginning of period \( t \), the representative agent’s problem writes:

\[
\max_{s_t, m_t, c_{t+1}} U(c_{t+1}, P_{t+1})
\]

subject to,

\[
\begin{align*}
    w_t + q_t E_t &= s_t + m_t \\
    c_{t+1} &= R_{t+1} s_t \\
    P_{t+1} &= P_t - \Gamma(P_t) + E_t - \gamma m_t \\
    m_t &\geq 0.
\end{align*}
\]

The first order condition reads:

\[
-R_{t+1} U_1(c_{t+1}, P_{t+1}) - \gamma U_2(c_{t+1}, P_{t+1}) + \mu = 0
\]

with \( \mu \geq 0 \), the associated Lagrange multiplier that satisfies:

\[
\mu m_t = 0.
\]

Since there is a non-negativity constraint on \( m_t \), we have to distinguish the case where agents choose not to engage in maintenance i.e., \( m_t \equiv 0 \), from the one where maintenance is operative i.e., \( m_t \geq 0 \). Moreover, equilibrium properties will depend on whether or not, the economy has achieved the irreversibility threshold \( \bar{P} \). These two elements define two distinct frontiers that divide the space of state variables \( k - P \) into four different sub-spaces. Therefore, the equilibrium analysis consists in studying separately the four regions for which the model exhibits quite distinct economic and environmental dynamics.

### 2.4 The frontier cases

We provide here a general definition of the frontiers. Next, a brief discussion is conducted on the issue of the admissibility of equilibria.

**Definition 1** In the \( k - P \) space, the first frontier, delimiting irreversible pollution levels from reversible ones, corresponds to the irreversibility threshold: \( P_t = \bar{P} \). The second frontier, thereafter called the indifference frontier, represents the set of points \( (k_t, P_t) \) where the agents are indifferent to whether or not they abate pollution. Let \( P_t = f(k_t, E_t) \) be this frontier. When the system is located in the region above the frontier (resp. below), maintenance is non-negative: \( m_t > 0 \) (resp. \( m_t \equiv 0 \)).
The indifference frontier is implicitly given by the FOC (12) in which we set \( m_t = \mu = 0 \). It defines the pollution as a monotonic decreasing function of both the capital stock and the quota: 
\[
P_t = f(k_t, E).
\]

Analyzing the admissibility problem boils down to investigating the location of different equilibria with respect to the two frontiers separating, on the one hand, the positive maintenance (PM) space \((m_t > 0)\) from the zero maintenance (ZM) area \((m_t = 0)\) and, on the other hand, the irreversible pollution space from the reversible zone. The next section addresses the existence of equilibria in the four regions of the \( k - P \) space. Now, assume that, for each dynamical system, there exists at least one stable steady state. Admissibility refers to the following reasoning: it is possible, during the convergence toward a stable solution of a determined zone, that the equilibrium path crosses one or the other frontier before reaching the steady state. But as soon as the trajectory goes through a frontier, the system is governed by new dynamics totally different from the ones valid in the previous region. In other words, the stable solution in consideration is not admissible since, once the frontier is crossed, the economy will converge to another stable solution associated with the new significant dynamics. The admissibility analysis is postponed to section 4.1 where some specific functional forms will be used.

In the following sections, our purpose is to evaluate the impact of environmental policy and the effect of a policy reform \((i.e.\ a\ change\ in\ the\ global\ quota\ \bar{E}_t)\) on equilibrium properties both at steady state and on the transitional dynamics. Assumptions 1 and 2 are assumed to hold throughout our analysis.

### 3 The competitive equilibrium

This section is devoted to the equilibrium analysis. We first focus on the properties (existence, uniqueness, stability) of each type of equilibrium by paying particular attention to the emergence of environmental and/or poverty traps as possible outcomes of equilibrium dynamics. Having assessed the cases where these undesirable states exist, we attempt to provide sufficient conditions to avoid them. By opposition to the traps, the emphasis is also put on the so-called desirable equilibria in the sense that they exhibit reversible pollution and important wealth. In the last part of the section, our aim will be to stress on the sensitivity of these desirable equilibria levels of capital and pollution with respect to a change in the quota.

Before proceeding with the analysis, we define mathematically the main concepts of interest to our future discussion.

We shall call a \textit{trajectory} of the economy a sequence of per capita variables \( \{c_t, m_t, s_t\} \), of aggregate variables \( \{L_t, K_t, E_t, P_t\} \) and of prices \( \{R_t, w_t, q_t\} \).

Next we define the intertemporal equilibrium with perfect foresight:

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4See section 4.1 for a complete examination of its properties.
Definition 2: Given the environmental policy \{\bar{E}_t\}, a competitive equilibrium is a trajectory of the economy such that:

i/ households and firms are at their optimum: the FOC (20) and the three conditions (6), (7) and (8), for profit maximization, are satisfied,

ii/ all markets clear: \( L_t = N = 1, K_{t+1} = s_t(= k_{t+1}) \) and \( E_t = \bar{E}_t(= e_t) \) on the permits market,

iii/ budget constraints (9) and (10) are satisfied,

iv/ the dynamics of pollution are given by (11).

In the sequel, equilibrium dynamics are derived from this definition in both the ZM and the PM region. Computing the dynamics is a necessary exercise in order to address the existence conditions of stationary equilibria belonging to each regions. Among the steady states, some have the features of environmental and/or poverty traps that can be summarized as follows:

Definition 3 (Environmental Trap (ET)): Given the environmental policy \{\bar{E}_t\}, an environmental trap is a trajectory of the economy such that:

\[
\lim_{t \to \infty} P_t = P_\infty ,
\]

with \( \bar{P} < P_\infty < +\infty \).

In other words, an environmental trap is a steady state that exhibits an irreversible level of pollution.

Definition 4 (Poverty Trap (PT)): Suppose multiple equilibria exist. Then, a trajectory that leads to the steady state with the lowest level of capital is called a poverty trap.

It is worth noting that a distinction is made between this kind of equilibria and what we call asymptotic traps. The asymptotic traps correspond to equilibrium trajectories accompanied by a perpetual erosion in environmental and/or economic resources:

Definition 5 (Asymptotic Environmental Trap (AET)): Given the environmental policy \{\bar{E}_t\}, an asymptotic environmental trap is a trajectory of the economy such that:

\[
\lim_{t \to \infty} P_t = + \infty .
\]

Definition 6 (Asymptotic Poverty Trap (APT)): Given the environmental policy \{\bar{E}_t\}, an asymptotic poverty trap is a trajectory of the economy such that:

\[
\lim_{t \to \infty} K_t = 0 .
\]

In the analysis to follow, we shall exclude the case where the variable \( K_t \) takes the value 0, since this case is not economically relevant.
3.1 Zero maintenance equilibrium (ZME)

The first region is the one where the constraint \( m_t \equiv 0 \) holds. It corresponds to the situation where the weight of environmental and financial constraints are such that households have not enough incentives to abate pollution. Thus, they devote the whole of their income \( \Omega_t \) to savings:

\[
\Omega_t = s_t
\]

with,

\[
\Omega_t = w_t + q_t E_t.
\]

This equality is substituting for the FOC (12).

From (6), (8) and the equilibrium condition for the permits market, we get the income as a function of \( k_t \) and \( \bar{E}_t \):

\[
\Omega_t = \Omega(k_t, \bar{E}_t) := (1 - \alpha) A k_t^\alpha \bar{E}_t^{1-\alpha-\beta}.
\]

Equilibrium dynamics directly follow from the combination of (11), (14), (15) and the market clearing condition for capital:

\[
\begin{align*}
{k_{t+1} = (1 - \alpha) A k_t^\alpha \bar{E}_t^{1-\alpha-\beta}} \\
{P_{t+1} = P_t - \Gamma(P_t) + \bar{E}_t}
\end{align*}
\]

and we shall note that stock variables dynamics are independent from each other. In addition, pollution accumulation is solely determined by the exogenous quota.

In this region of the \( k - P \) space, a steady state (thereafter SS) solves:

\[
\begin{align*}
{k = (1 - \alpha) A k^\alpha \bar{E}^{1-\alpha-\beta}} \\
{\Gamma(P) = \bar{E}}
\end{align*}
\]

where \( \bar{E} = \lim_{t \to \infty} E_t \) is assumed to exist.

The first equation admits a unique solution different from 0:\(^5\) \( k^*_c(\bar{E}) = \bar{k}(\bar{E}) \) with,

\[
\bar{k}(\bar{E}) = \left[(1 - \alpha) A \bar{E}^{1-\alpha-\beta}\right]^{\frac{1}{1-\alpha}}
\]

and the existence conditions are summarized in the following proposition:

**Proposition 1** For the dynamics (16):

1/ there is no environmental trap but there always exists an asymptotic environmental trap;

---

\(^5\)The subscript "c" (resp. "i") stands for the corner or zero maintenance (resp. interior or positive maintenance) solution. The second index "i" (resp. "i") will refer, in the remainder of the paper, to a reversible (resp. irreversible) level of pollution.
ii/ there exists a steady state with a reversible level of pollution if and only if
\[ \max_{P \in [0, \tilde{P}]} \{ \Gamma(P) \} = \Gamma_{\max} \geq \tilde{E}. \]  

If \( \Gamma_{\max} > \tilde{E} \), then there exist two distinct steady states, \( (k_{cr}^*, P_{cr}^-) \) and \( (k_{cr}^*, P_{cr}^+) \), that are ordered as:
\[ P_{cr}^- \leq \tilde{P} \leq P_{cr}^+ < \tilde{P}. \]

The steady state \( (k_{cr}^*, P_{cr}^-) \) is locally stable, the other one is unstable.

**Proof.** When pollution is irreversible, the second equation in (17) imposes \( \tilde{E} = 0 \). But, under our specifications of production and preferences (Assumption 2), we can exclude this limit case. In order to prove the existence of an asymptotic environmental trap, it is enough to see that if, for some \( t_0 \), \( P_{t_0} > \tilde{P} \), then the dynamics of pollution (16) write: \( P_{t+1} = P_t + \dot{E}_t \), for all \( t \geq t_0 \), and therefore if \( \dot{E}_t \to \bar{E} \), the sequence \( P_t \) diverges to infinity.

If pollution is reversible, then the stationary level of pollution must solve, for a given quota \( \bar{E} \): \( \Gamma(P) = \bar{E} \). According to the inverted U shape of the function \( \Gamma(P) \), it is clear that \( \Gamma(P) = \bar{E} \) admits a solution \( P_{cr}^*(\bar{E}) \) iff \( \Gamma_{\max} \geq \bar{E} \) where \( \Gamma_{\max} \) is the maximal absorption level, reached for a given \( \bar{P} \). Moreover if the inequality in (19) is strict, then we get two positive steady state values for pollution: the fact that \( \Gamma() \) is increasing, then decreasing, is enough to guarantee that exactly two steady states exist. The proof of stability properties is provided in appendix B.1. ■

Condition (19) was already used in Talvonen and Withagen [1996] or Prieur [2007] and conveys the idea that the maximum potential of assimilation by nature is higher than the stationary emissions level. The latter precisely corresponds, in the zero maintenance space, to the global quota on emissions. It is worth noting that the necessary and sufficient condition (19) imposes an upper bound to the domain of variation of \( \bar{E} \).

Assume the economy has a unique instrument to control pollution, the pollution permits. Then what happens is the following: when the quota is set below the upper bound \( \Gamma_{\max} \), two scenarios may occur depending on its initial location in the space \( k - P \). An economy relatively little polluted will reach the desirable SS. Even if the productive activity generates emissions, it benefits from an increasing natural assimilation and the pollution stock remains below the threshold \( \tilde{P} \) until the SS is hit. Conversely, the economy would likely suffer from an irreversible degradation of the environment if it starts from a highly polluted state. Indeed, the higher the initial pollution, the earlier the assimilation capacity is exhausted. After the threshold is exceeded, the logic of the AET applies since pollution stock indefinitely accumulates.

When permits are the only means at the economy’s disposal, the situation may be even worse if the quota is fixed above the bound \( \Gamma_{\max} \). In this case, there no wealthy SS with reversible pollution and the economy is doomed to follow the trajectory of the AET.

This observation highlights the importance of introducing the second instrument, that is, the
environmental maintenance. With maintenance, the AET does not matter anymore since this trajectory will necessarily cross the indifference frontier and then reach the positive maintenance subspace. In other words, once maintenance is taken into account, the AET appears to be an inadmissible equilibrium trajectory.

Studying the dynamics of the positive maintenance region is precisely the purpose of the next section.

3.2 Positive maintenance equilibrium (PME)

The region with \( m_t > 0 \) refers to situations where the economy is relatively wealthy but suffers from harmful pollution. Consequently, agents are willing to engage in maintenance. In this case, the households’ problem admits an interior solution and the FOC is:

\[
-R_{t+1}U_1(c_{t+1}, P_{t+1}) - \gamma U_2(c_{t+1}, P_{t+1}) = 0. \tag{20}
\]

The equilibrium analysis consists in considering the system of equations (6)-(8), (11), (20) and the market clearing conditions. Combining these equations yields the expression of consumption and maintenance decisions as a function of the capital stock and the quota:

\[
c_t = \alpha A k_t^\alpha \bar{E}_t^{1-\alpha - \beta} \tag{21}
\]

\[
m_t = (1 - \alpha) A k_t^\alpha \bar{E}_t^{1-\alpha - \beta} - k_{t+1}. \tag{22}
\]

By substituting expressions (7) and (21) into the FOC, equation (20) rewrites:

\[
-R(k_{t+1}, \bar{E}_{t+1})U_1(c(k_{t+1}, \bar{E}_{t+1}), P_{t+1}) - \gamma U_2(c(k_{t+1}, \bar{E}_{t+1}), P_{t+1}) = 0. \tag{23}
\]

This equation implicitly defines an equilibrium relation, valid for any \( t \), between \( P_t, k_t \) and \( \bar{E}_t \):

\[
P_t = \Phi(k_t, \bar{E}_t) \tag{24}
\]

that governs the dynamics in the whole positive maintenance (PM) space.

Under Assumption 2, the function \( \Phi() \) is decreasing in \( k_t \): \( \Phi_1 < 0 \). A rise in \( k_t \) tends to reduce the cost of maintenance (first term in (23)) since it lowers the interest factor and increases the consumption \((R_1 < 0, c_1 > 0, U_{11} < 0)\). In addition, due to the distaste effect exerted by pollution and the rise in consumption, it also goes with an increase in the benefits arising from maintenance \((c_1 > 0, U_{12} < 0)\). Therefore, the higher the capital, the higher the incentive to maintain and the lower the pollution. According to this relation, at each period, capital stock is

\(^{6}\)Total differentiation of (23) with respect to \( k \) yields:

\[
\frac{dP_t}{dk_t} = -\frac{R_t U_1 + R_{C1} U_{11} + \gamma c_1 U_{12}}{RU_{12} + \gamma U_{22}}.
\]

This derivative is negative under the assumptions on preferences and the Cobb-Douglas production function.
inversely linked to the pollution concentration. Note that the sign of \( \Phi_2 \) is a priori indeterminate. If a rise in \( \bar{E}_t \) increases the benefits to maintain (through the increase in \( c() \)), it is associated with two opposite effects on the cost of maintenance. In fact, it rises both the interest factor and the consumption. Now, if we assume that the intertemporal elasticity of substitution \( \sigma = -U_1/(cU_{11}) \) is lower than one, which means that savings is decreasing in the interest factor, the overall effect on the cost is negative. Finally, we have \( \Phi_2 < 0 \).

Dynamics are then described by the following system of equations,

\[
\begin{align*}
    P_{t+1} &= \Phi(k_{t+1}, \bar{E}_{t+1}) \\
    P_{t+1} &= P_t - \Gamma(P_t) + \Theta(k_t, \bar{E}_t, k_{t+1})
\end{align*}
\]

(25)

where \( \Theta(k_t, \bar{E}_t, k_{t+1}) \) represents the real emissions:

\[
\Theta(k_t, \bar{E}_t, k_{t+1}) = \bar{E}_t - \gamma \left( (1 - \alpha) A k_t^\alpha \bar{E}_t^{1-\alpha-\beta} - k_{t+1} \right)
\]

Let us now turn to conditions for the existence of stationary equilibria and asymptotic traps as possible equilibrium outcomes. As for steady states, in a first step, we do not ask whether these equilibria exhibit reversible or irreversible pollution. Note also that we restrict the study to the interval \([0, \bar{k}(\bar{E})]\) (with \( \bar{k}(\bar{E}) \) defined in (18)) on which stationary maintenance is necessarily non-negative (see appendix A.1). At a SS, the system to solve becomes:

\[
\begin{align*}
    P &= \Phi(k, \bar{E}) \\
    \Gamma(P) &= \Theta(k, \bar{E}, k),
\end{align*}
\]

where \( \Gamma(P) \) may be zero if \( P \) happens to be larger than \( \bar{P} \). Note that the study (existence, stability) of positive maintenance steady states (PMSS) will be performed for a given \( \bar{E} \). It allows us to rewrite functions \( \Phi(k, \bar{E}) \) and \( \Theta(k, \bar{E}, k) \) as follows:

\[
\Phi(k, \bar{E}) = \varphi(k)
\]

with \( \varphi'(k) < 0 \), and

\[
\Theta(k, \bar{E}, k) = \theta(k).
\]

Therefore, the problem of finding equilibria is reduced to the analysis of equation:

\[
\Gamma(\varphi(k)) = \theta(k).
\]

(27)

This is done in appendix A.3, where the following result is proved.

---

\(^7\)The expression of \( \Phi_2 \) is given by:

\[
\frac{dP_t}{d\bar{E}_t} = -\frac{R_2U_1 + R_2U_{11} + \gamma c_2 U_{12}}{RU_{12} + \gamma U_{22}}
\]

The sign of the numerator in \( \Phi_2 \) is unknown. But, we can rewrite:

\[
R_2U_1 + R_2U_{11} = R_3U_1(1 - \frac{1}{\sigma})
\]

now, imposing \( \sigma < 1 \) implies \( \Phi_2 < 0 \).
Proposition 2 For the dynamics given by (25):

i/ A necessary condition for an environmental and poverty trap to exist, is that the quota be below the limit value \( E_L \), with:

\[
E_L = \left( \gamma A(1 - \alpha)^2 \right)^{\frac{1 - \alpha}{1 - \beta}} (\alpha(1 - \alpha)A)^{\frac{1}{2}}.
\]  

(28)

The dynamics (25) admit at most two steady states (but only one locally stable) with irreversible pollution.

ii/ Assume that, \( E \) being given, the function \( \varphi(k) = \Phi(k, E) \) is such that

\[
\lim_{k \to 0} \varphi(k) = + \infty.
\]

Then there exists an asymptotic poverty and environmental trap for the dynamics (25).

iii/ If (19) holds, that is:

\[
\max_{P \in [0, \bar{P}]} \{ \Gamma(P) \} = \Gamma_{\text{max}} \geq \bar{E},
\]

and if

\[
\bar{k}(\bar{E}) \geq \varphi^{-1}(P_{\text{cr}}^{+})
\]

(29)

then there exists a locally stable steady state \((k_{ir}^+, \bar{E}), P_{ir}^+(\bar{E})\) with reversible pollution.

Proof. The proof is in appendix A.2, A.3 and B.2. ■

The statement above is, on purpose, relatively imprecise with respect to the exact number of equilibria. Studying the existence of PMSS turns out to be a tedious exercise. Therefore, in proposition 2, we choose to focus on the most relevant SS from the point of view of our analysis. They comprise, on the one hand, the asymptotic and stationary traps (our first aim being to identify conditions ensuring the avoidance of such undesirable states) and, on the other hand, the stable SS with reversible pollution (since the emphasis will be next put on the impact of a change in the policy on desirable equilibria properties).

However, it is possible to provide a more general characterization of PMSS by referring to a simple graphical analysis (see appendix A.3).

Despite the permit regime, the economy may reach a SS with irreversible pollution. The environmental trap also corresponds to a poverty trap since, according to (24), it exhibits a level of capital less than the one reached at any PM solution with reversible pollution.

The important point is that environmental poverty generates economic poverty. The mechanism behind the emergence of this long term state is discussed in detail in Priour [2007] and can be summarized as follows. The economy is located in a region with a pollution level above the threshold \( \bar{P} \). At the same time, the policy is so stringent that the level of wealth is also very low. In this context, the following logic applies. The environmental pressure forces agents to devote a
sizeable share of resources to maintenance. But, this effort is done at the expense of productive savings and causes a break in capital accumulation. Combining with the low level of wealth, this in turn implies that the opportunity cost of maintenance becomes more and more severe. Finally, the economy fails to artificially restore environmental quality. Indeed the reaction of the agents to environmental degradation only allows the economy to stabilize pollution at a constant - but higher than $P$ - level. However this steady state is reached at the expense of wealth.

Contrary to the previous study, it is possible to impose a condition that prevents from the occurrence of such a long term state.

**Corollary 1** A sufficient condition to exclude the existence of environmental and poverty traps is to fix the quota on emissions to a sufficiently high level: $\bar{E} > \bar{E}_L$.

Thus the environmental policy, enacted at the supranational scale, should authorize firms to emit a sufficient amount of pollution to avoid the economy possible stabilization in a trap.

This *a priori* surprising result is explained by the fact that, in the positive maintenance space, the economy has two instruments that affect the level of polluting emissions. Now, the existence of a steady state supposes that real emissions are nil. In other words, pollution abatement by households must exactly compensate for polluting emissions by firms. This situation precisely occurs when the exogenous quota is set below the critical value $\bar{E}_L$. By fixing $\bar{E} > \bar{E}_L$, one mechanically ensures the absence of $E&PT$ but the scope of this result must be relativized.

In fact, it remains two possible outcomes. Once again, depending its initial location in the PM space, the economy may follow two opposite development trajectories. The first one leads to the desirable PMSS, defined in iii/, that exhibits the lowest level of pollution. Since, at equilibrium, pollution and capital are inversely linked, it is also associated with the highest wealth.\(^8\) This SS exists under a condition similar to the necessary and sufficient condition (19) used to prove the existence of zero maintenance steady states (ZMSS). Existence also requires an additional sufficient condition (29) ensuring some correspondence between the domains of variation of the stock variables $k$ and $P$.

The second trajectory has the features of both an asymptotic environmental and poverty trap, $AE&PT$. Such an undesirable outcome exists under a very general condition involving preferences.\(^9\) Now by fixing the quota above $\bar{E}_L$, it is highly likely that the economy, located in the region with irreversible pollution, finally suffers the perpetual increase in pollution associated with an continuous erosion of the level of wealth that entails the $AE&PT$.

So, the question that immediately comes into mind is the following: what is the best option from the point of view of the policy maker? to exclude stationary traps by imposing $\bar{E} > \bar{E}_L$ provided that it supposes to run a higher risk to be trapped into the $AE&PT$. Or to keep the

\(^8\)That is why we use the superscript "−" to reflect the level of pollution and "+" for the level of capital.

\(^9\)Which is notably fulfilled by the standard utility function used in section 4.1. It is logarithmic in consumption and quadratic in pollution.
$E&PT$, that appears to be a lesser evil compared to the $AE&PT$, since the economy, located in this region, would have a greater opportunity to reach it rather than to diverge.

A way of answering this question\textsuperscript{10} requires to address two additional but related questions: Can an economy, that does not initially belong to the irreversible region, reach it and diverge? To what extend does the possible divergence depend on the choice of the quota?

Assuming $\bar{E} > \bar{E}_L$, the answer involves to assess the impact of a change in the quota $\bar{E}$ on the frontier delimiting:
- the basin of attraction of the PMSS $(k_{ir}^+ (\bar{E}), P_{ir}^-(\bar{E}))$
- the set of points from where the economy follows the $AE&PT$.

Actually if there exists a rule, concerning the choice of $\bar{E}$, that prevents the economy with initial reversible pollution to diverge, then the recommendation will be to set $\bar{E} > \bar{E}_L$.

We will go back to this important point in section 4.2 devoted to the dynamic analysis.

3.3 Admissibility analysis: general discussion

Admissibility analysis consists in checking whether the steady states found for each of the four dynamical systems, are indeed located within the domain of definition of this particular dynamical system. We shall make the convention that a potential equilibrium is admissible if and only if it lies in the interior of the domain.

Proving the admissibility of SS with respect to the first frontier (or irreversibility threshold, see Definition 1) is straightforward. The way we define and characterize ZM and PMSS ensures they belong to the right domain. For instance, according to the existence condition (19) in proposition 1, reversible ZMSS necessarily exhibit a pollution concentration below the threshold $P$.

For the second frontier (the indifference frontier), it proves difficult to state simple conditions for admissibility or non-admissibility in the general case. This is why the question of admissibility will be treated in detail in a particular case in Section 4.1.

There is however an important property which can be proved under quite general assumptions.

**Proposition 3** Assume that there is one stable ZME, $(k_{ir}^*, P_{ir}^*)$. Then:

i) There exists a stable PME in the sense of Proposition 2 iii), if and only if the ZME is not admissible.

ii) If there exists a stable PME in the sense of Proposition 2 iii), then this equilibrium is admissible.

**Proof.** First observe that according to Proposition 1, Condition (19) is a necessary condition for a stable ZME to exist. From the analysis of Section 3.1 (see also Appendix B.1), it results the

\textsuperscript{10}That is influenced by our aim to provide sufficient conditions to avoid traps.
ZME \((k^*_c, P^*_c)\) is solution of the fixed-point system:

\[
k = \Omega(k), \quad P = g(P) := P - \Gamma(P) + \bar{E}.
\]

According to (22) and (25), the indifference frontier is the set of points \((k_t, P_t)\) which satisfy \(m_t = 0\), that is:

\[
P_{t+1} = \varphi(k_{t+1}) \quad P_{t+1} = g(P_t) \quad k_{t+1} = \Omega(k_t).
\]

In other words, this is the set of points which obey simultaneously the two dynamics. The equation of the frontier is therefore:

\[
g(P) = \varphi(\Omega(k)).
\]

More precisely, the zero-maintenance region is the set of points where \(\varphi(\Omega(k)) \geq g(P)\). The interior of this region is characterized by the inequality \(\varphi(\Omega(k)) > g(P)\), and the positive maintenance region is where \(m_t > 0\), which is equivalent to \(\varphi(\Omega(k)) < g(P)\). We have already seen that under Assumption 2, the function \(\varphi(k)\) is decreasing.

Assume now that the ZME \((k^*_c, P^*_c)\) is admissible, that is, in the ZM region. Necessarily,

\[
\varphi(\Omega(k^*_c)) > g(P^*_c).
\]

But \(k^*_c = \Omega(k^*_c)\), and \(P^*_c = g(P^*_c)\) so the condition is simply:

\[
\varphi(k^*_c) > P^*_c.
\]

The condition (29) of Proposition 2 \(iii\) for the existence of a locally stable PME is:

\[
\varphi^{-1}(P^*_c) \leq k^*_c,
\]

which is equivalent to:

\[
P^*_c \geq \varphi(k^*_c).
\]

This is in contradiction with Condition (30). This PME does not exist. Conversely, if the inequality (29) holds, the PME exists, but the ZME is not admissible. This proves statement \(i\).

Assume now that (29) holds, and that the stable PME exists. The analysis of Appendix A.3 explains that the capital level \(k^*_i + \) solves the equation:

\[
\Gamma(\varphi(k_{ir})) = \bar{E} - \gamma m(k_{ir}).
\]

On the one hand, since the equilibrium maintenance is positive, we must have \(m(k_{ir}) = \Omega(k_{ir}) - k_{ir} > 0\). Since \(\varphi\) is decreasing, this implies:

\[
\varphi(\Omega(k_{ir})) < \varphi(k_{ir}).
\]
On the other hand, introducing the functions $g(\cdot)$ and $\Omega(\cdot)$ in (31), we find:

$$\varphi(k_{ir}) = g(\varphi(k_{ir})) - \gamma m(k_{ir}) < g(\varphi(k_{ir})).$$  \hspace{1cm} (33)

Joining (32) to (33) and using the fact that $P_{ir} = \varphi(k_{ir})$, we find:

$$\varphi(\Omega(k_{ir})) < g(P_{ir}),$$

which precisely means that the PM equilibrium is admissible. ■

To conclude this admissibility analysis, observe that some cases are left open by Proposition 3. Indeed, the proposition does not exclude the existence of stable and admissible PME, which would coexist with the stable and admissible ZME.

3.4 Comparative Statics: Impact of a political reform

Once the existence conditions are set, we focus on the effect of an environmental policy reform in both desirable ZM and PM stationary equilibria. Note that our approach only makes sense if we restrict the analysis to the (locally) stable equilibria: the ones with the lowest level of pollution, according to proposition 1, ii/ and to proposition 2, iii/. We first consider repercussions of a strengthening in the permits system, consisting of a decrease in the emission quota, on the stable ZME.

**Proposition 4** A decrease in the quota $\bar{E}$ implies a fall in both the levels of pollution and capital at the stable ZMSS.

**Proof.** It is straightforward to see that $k_{ir}'(\bar{E}) > 0$. Now, if we refer to the inverted-U shape of the assimilation function $\Gamma(P)$, then it is clear that the fall in the quota $\bar{E}$ causes a fall in the level of stationary pollution at the low stable steady state: $P_{ir}'(\bar{E}) > 0$. ■

When only one of the two instruments for pollution control (permits) is operative, we find a result close to Jouvet, Michel and Vidal [2002]'s conclusion, obtained for complementary inputs, in the case where they are substitutable (the elasticity of substitution is equal to one for a Cobb-Douglas technology).\(^{11}\) In fact, we detect the existence of a dilemma between economic growth and environmental preservation: a stricter policy means a lower level of stationary pollution but it is done at the expense of capital accumulation and long term wealth. A reduction of $\bar{E}$ causes a drop in both emissions and the pollution accumulated at each period. However, it also generates a negative income effect (see the budget or financial constraint (9)). A lower quota means a reduction of the wage and the environmental allowance which implies that the agent has relatively less resources to devote to savings and maintenance (tightening of the financial

\(^{11}\)Note that the authors consider a framework that is quite distinct from ours. They assume that infinite lifetime permits belong to households that pass them on from generations to generations and rent them to firms. Thus comparing our results with theirs is a purely informal exercise but is explained by the closeness of our problematics.
constraint). Thus, this income effect translates into a slower capital accumulation and a lower stock of capital at the ZMSS.

Next, the same analysis is conducted for the PMSS. It yields the following result:

**Proposition 5** At the stable PMSS \((k_{ir}^{*+}(\bar{E}), P_{ir}^{*-}(\bar{E}))\):

i/ if

\[
\bar{E} \geq (\gamma(1 - \alpha - \beta))^{(1-\alpha)/\beta} (A(1 - \alpha))^{1/\beta}
\]

then \(k_{ir}^{*+}(\bar{E}) < 0\),

ii/ if, in addition,\(^{12}\)

\[
\frac{\Theta_1(k_{ir}^{*+}(\bar{E}), \bar{E}, k_{ir}^{*+}(\bar{E}))}{\Theta_2(k_{ir}^{*+}(\bar{E}), \bar{E}, k_{ir}^{*+}(\bar{E}))} < \frac{c_1(k_{ir}^{*+}(\bar{E}), \bar{E})}{c_2(k_{ir}^{*+}(\bar{E}), \bar{E})}
\]

then \(P_{ir}^{*-}(\bar{E}) > 0\).

The proof is provided in appendix C.

Increasing \(\bar{E}\) has two opposite effects on real emissions. First, it entails a rise in polluting emissions by firms. But, it also stimulates maintenance, through the positive income effect, which in turn tends to reduce emissions. Now, under condition (34), the net effect is positive, that is, real emissions are increasing in \(\bar{E}\) at equilibrium.

This condition is sufficient to show that a reduction of the quota causes a rise in the stock of capital at the stable steady state. Let us decompose the effect of a fall in the quota on stationary variables. This decrease is still associated with a negative income effect described above. But there is now an additional substitution effect. This effect is mainly due to the fall in emissions and the pollution accumulated at each period (see the dynamics given by (1)). *Ceteris paribus*, with the reduction of the quota, the affected generation can allocate a lower amount of resources to maintain environmental quality which will be enjoyed in second period of life (slackening of environmental constraint). It also allows the households to save a bigger share of their income which favours capital accumulation.

Now, we have to address the question to know why the latter effect dominates at the stable PMSS. At this SS, before the reform, the economy is endowed with an important capital stock. Moreover, pollution level is less than the threshold \(\bar{P}\) and is accompanied by an assimilation function that is increasing in the stock of pollutant. The fall in the quota causes a decrease in the income which is a priori unfavourable to both savings and maintenance expenditures. But, this

\(^{12}\)With a slight abuse of notation, we have denoted:

\[
\Theta_k = \frac{d}{dk} \Theta(k, \bar{E}, k).
\]
tightening of the financial constraint remains quite moderate since the economy owns a sizeable level of wealth. The reduction of the amount of permits sold to firms also implies a slackening of the environmental constraint that was already not very stringent. Therefore, the agent has some latitude to absorb the repercussions of the income decrease on capital accumulation. Here the substitution effect is entirely applicable: maintenance serves as an adjustment variable in such a way that the fall in income does not penalize savings. Finally, the level of capital raises.

The second condition (35) concerns the direct and indirect effects of a change in $\bar{E}$ on both real emissions and consumption. An increase in $\bar{E}$ rises consumption through its (direct) positive effect on the interest factor. However, it causes a drop in capital (since $k_{1r}(\bar{E}) < 0$) that, on the contrary, lowers consumption (indirect negative effect). The same reasoning applies for real emissions. If a higher quota means higher emissions (direct positive effect), it is also associated with a lesser capital and, consequently, lesser emissions (indirect negative effect). This condition finally states that the ratio of the effects on consumption exceeds the corresponding ratio for emissions. This inequality holds if, for instance, the global impact of a rise in $\bar{E}$ on consumption is negative while it is positive for emissions (see appendix C) and we have $P_{rr}(\bar{E}) > 0$. In this case, a reduction of the quota first implies a fall in emissions that tends to reduce the level of pollution at the stable SS. Second, it results in an increase in equilibrium consumption that goes with a decrease in pollution because of the disutility effect exerted by pollution.

Therefore, the reform procures a double dividend since a more restricting quota allows the economy to reach a steady state where both the level of wealth is higher and the pollution is lesser. The impact of a fall in $\bar{E}$ presents some similarities with the conclusions of the literature on tax reform (see among others, Bovenberg and Smulders [1995], [1996] or Bovenberg and de Mooij [1997]). We shall also note that our result, contrary to the aforementioned papers, is not conditioned by the controversial assumption of the existence of a positive environmental externality in production.

In this section, we have shown the existence of multiple equilibria with very different properties. As in Prieur [2007], some are similar to environmental and poverty traps. However, in our setting, it appears that it is possible to prevent the economy from stabilizing at such a state provided that the quota is set at a sufficiently high level. The analysis of the impact of an environmental policy reform on different equilibria properties reveals two important features. First, in the absence of maintenance by households, a stricter policy allows to reach a less polluted long run state but is detrimental to capital accumulation. Second, in the PM space, it brings a double dividend.

The following section deepens the dynamic analysis by first studying the location and the evolution of the frontiers with respect to the quota. Then, it deals with the issue of the admissibility

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Note that if, following Ono [2002], we consider the specific class of separable utility functions that are logarithmic in consumption, we necessarily have $P_{rr}(\bar{E}) > 0$ under condition (34) alone.
of the different equilibria. Finally, it goes back to the problem of divergence (or, equivalently, convergence to an AE&PT) in a numerical example. It is worth noting that most the analysis we be conducted under the use of specific utility and assimilation functions. This approach allows us not only to explicitly study the admissibility of steady states but also to compute the global dynamics so as to perform the simulations.

4 Effect of permits on dynamics

4.1 Admissibility analysis: a special case

Let us focus on the properties of the indifference frontier. In order to simplify this analysis and simply draw a tendency concerning the frontier' behaviour relatively to the quota, we assume a constant environmental policy i.e. $E_t = \bar{E}$ for $t$.

When assuming $m_t = \mu = 0$ in (12), the FOC rewrites:

$$-R(\Omega(k_t, \bar{E})U_1(R(\Omega(k_t, \bar{E}))\Omega(k_t, \bar{E}), P_t - \Gamma(P_t) + \bar{E}) - \gamma U_2(R(\Omega(k_t, \bar{E}))\Omega(k_t, \bar{E}), P_t - \Gamma(P_t) + \bar{E}) = 0$$

(36)

and implicitly defines the indifference frontier. Total differentiation of (36) then gives:

$$\frac{dP_t}{dk_t} = -\Omega_1 \frac{R_1 U_1 + (RU_{11} + \gamma U_{12})((R_1 \Omega_2 + R_2)\Omega + R\Omega_2)}{(1 - \Gamma')(RU_{12} + \gamma U_{22})}$$

$$\frac{dP_t}{dE} = -\frac{(R_1 \Omega_2 + R_2)U_1 + (RU_{11} + \gamma U_{12})((R_1 \Omega_2 + R_2)\Omega + R\Omega_2)}{(1 - \Gamma')(RU_{12} + \gamma U_{22})}$$

and, under our assumptions on preferences, the Cobb-Douglas technology and the condition on the elasticity of substitution, these derivatives are negative. The richer the economy, the lower the level of pollution from which the decision to abate pollution is taken.\textsuperscript{14} Moreover, we note that the frontier falls, in the space of states variables, when the quota increases.

Starting from an initial state in the ZM zone, the economy will engage all the faster in maintenance since the global quota is high. This feature has non trivial implications on the agents’ behaviour. In fact, emitting an important amount of permits will hasten the moment when the agents will be willing to devote resources to environmental maintenance. This property is directly due to the tightening of the environmental constraint: more permits allocated to firms means more harmful emissions at each period. The explanation is also based on the role played by the redistributive facet of environmental policy: a rise in $\bar{E}$ tends to increase the agents’ income and implies that the weight of the financial constraint diminishes in their tradeoffs. Thus, they have a greater incentive to reduce pollution. On the contrary, fixing the quota to a lower level provokes a shifting of the frontier toward the top of the $k - P$ space and delays the instant when maintenance becomes operative.

\textsuperscript{14}Which explains once again by the evolution of the balance of power between environmental and financial constraints.
One may ask the question to know whether or not it is better to set a high quota, so as to reach the indifference frontier rapidly but with a potentially important level of pollution, or to resort to a stringent policy with a low quota (which implies a more distant frontier and a slower accumulation of pollution). Part of the answer is provided by the admissibility analysis of the different equilibria.

From now on, we consider specific forms for the utility and the assimilation functions. More precisely, we use the following logistic specification of the assimilation, 

\[ \Gamma(P_t) = \begin{cases} 
\theta P_t (\bar{P} - P_t) & \forall P_t < \bar{P} \\
0 & P_t \geq \bar{P}
\end{cases} \]

and we assume an additive utility function:

\[ U(c_{t+1}, P_{t+1}) = \log c_{t+1} - \phi \frac{P^2_{t+1}}{2}. \]

The assimilation function \( \Gamma() \) defined above satisfies Assumption 1 if \( \theta < 1 \), with \( \bar{P} = \bar{P}/2 \) and \( \Gamma_{\max} = \theta \bar{P}^2/4. \) The utility function satisfies Assumption 2, and the function \( \Phi() \) defined by (23) and (24) is:

\[ \Phi(k, \bar{E}) = \frac{1}{\gamma k}. \]

This function does satisfy the assumption \( ii) \) in Proposition 2.

Let us define the values \( \bar{k}(\bar{E}) \) and \( \hat{k}(\bar{E}) \) as follows:

\[ \bar{k}(\bar{E}) = \left( \frac{1}{\gamma \phi (P + \bar{E})(1 - \alpha)A \bar{E}^{1-\alpha-\beta}} \right)^{1/\alpha}, \]

\[ \hat{k}(\bar{E}) = \left( \frac{1}{\gamma \phi E (1 - \alpha)A \bar{E}^{1-\alpha-\beta}} \right)^{1/\beta}. \]

The indifference frontier, for the example considered here, is given by (see appendix D.1):

\[ f(k_t, \bar{E}) = \begin{cases} 
g(k_t, \bar{E}) & \text{for any } k_t \in [0, \bar{k}(\bar{E})] \\
\frac{1}{2\theta} \left( - (1 - \theta \bar{P}) + \sqrt{(1 - \theta \bar{P})^2 + 4\theta g(k_t, \bar{E})} \right) & \text{for any } k_t \in (\bar{k}(\bar{E}), \hat{k}(\bar{E})] \\
0 & \text{for any } k_t > \hat{k}(\bar{E})
\end{cases} \]

with,

\[ g(k_t, \bar{E}) = \frac{1}{\gamma \phi (1 - \alpha)A k_t^{\alpha} \bar{E}^{1-\alpha-\beta}} - \bar{E}. \]

Now we can focus on the admissibility of the different steady states. By definition, both ZM and PM reversible solutions satisfy the first admissibility condition, concerning their location with respect to the irreversibility threshold. The study of the location of the two types of SS with respect to the other frontier is explained in appendix D.2. The respect of the second admissibility
condition, simultaneously by ZM and PM solutions, imposes that the quota belongs to a specific range: $\bar{E} \in [\bar{E}_i, \bar{E}_s]$. The upper bound refers to the location of stationary capital, for a ZMSS, with regard to the striking value $\bar{k}(\bar{E})$. If the quota exceeds the amount $\bar{E}_s$, then the frontier cuts the absissa axis ($P_i = 0$) before the level $\bar{k}(\bar{E})$. Therefore, the economy finds itself unable to reach the ZM solutions since these last are finally located in the PM space and consequently are not admissible. Thus the inequality $k^*_r(\bar{E}) \leq \bar{k}(\bar{E})$, that can be rewritten $\bar{E} \leq \bar{E}_s$, is a necessary condition for admissibility of ZMSS.$^{15}$

It is worth noting that the ranking between the bounds of the range $[\bar{E}_i, \bar{E}_s]$ and $\bar{E}_L$ (defined by (28)) is a priori unknown. If $\bar{E}_L < \bar{E}_s$, then there exists a non empty range $(\bar{E}_L, \bar{E}_s)$ on which it is possible to fix a quota that avoids poverty traps and satisfies the necessary condition for admissibility of ZMSS. If, however, $\bar{E}_L \geq \bar{E}_s$, then we face the following trade-off: imposing a quota $\bar{E} > \bar{E}_L$ ensures the absence of traps but is also translated into the impossibility of reaching the ZM solutions. On the contrary, by choosing a quota such that $\bar{E} < \bar{E}_s$, the economy could potentially converge to the better or the worse SS. For PMSS, the same reasoning applies, in a symmetrical way, when we consider the ranking between $\bar{E}_i$ and $\bar{E}_L$.

Now we proceed to simulations so as to measure global dynamics’ adjustments with respect to a change in the quota.

4.2 Numerical Example

The study of the PME has shown that it is possible to exclude environmental and poverty traps provided that the exogenous quota is higher than the critical level $\bar{E}_L$ (see section 3.2). However, nothing guarantees that the growth path of the economy, located in the region with irreversible pollution, is not an asymptotic environmental and/or poverty trap (see figure 2 for an illustration of this kind of trajectory; see also appendix D.3 for more details)$^{16}$ In the sequel, we will indirectly speak of "convergence toward an asymptotic trap" or "divergence" since along this type of trajectory the sequence of pollution stocks diverges.

Starting the analysis by assuming that the quota equals a level $\bar{E} > \bar{E}_L$, we wonder what is the impact of the choice of $\bar{E}$ on the possibility, for the economy, to reach the irreversibility

---

$^{15}$The converse inequality $\bar{E} \geq \bar{E}_i$ is only a sufficient admissibility condition for PM solutions (see appendix D.2).

$^{16}$The thinking behind the existence of this kind of development path is the following. In this region, the pollutant concentration is such that, on the one hand, nature does not assimilate pollution any more and, on the other hand, households suffer from the damages caused by pollution. In order to remedy to these damages, they have no other option than to devote a sizeable share of their resources to maintenance. But this decision goes against savings and consumption (that must remain positive according to preferences). Therefore, it translated into a break in capital accumulation. Moreover, this effort reveals insufficient, on the duration, to compensate for polluting emissions by firms and to stop the rise in pollution. Even if, between the first and second periods, pollution decreases, we next see a fall in capital stock associated with an increase in pollution. This impoverishment process will inexorably reoccur, from periods to periods. The trajectory finally meets the equilibrium relation.
space\(^{17}\) knowing that it does not originally belong to it.

In order to deal with this issue, we compute the global dynamics of the model for the specifications given in section 4.1 (see appendix E) and make some simulations for the following set of parameters:

\[
\{A, \alpha, \beta, \theta, \gamma, \phi, \bar{P}\} = \{1.9, 0.3, 0.6, 0.15, 1, 1, 6\}.
\]

More precisely, we focus on the dynamics in the PM region and compare the evolution of the stable reversible solution's basin of attraction\(^{18}\) with the one of the divergence region. Since the irreversibility threshold equals \(E_L = 0.58\), the simulations are made for two distinct values of the global quota: \(E_1 = 0.6\) and \(E_2 = 1.2\). Graphical representations of the basins of attraction, for both of these values, are displayed in figures 3 and 4.

The comparison between these two graphics clearly reveals that the "frontier" delimiting the basin of attraction from the divergence region shifts down, in the \(k - P\) space, when the quota raises. Whereas diverging requires to be initially situated in the irreversible pollution region when the quota is low (except for very low capital levels), we see that the set of initial conditions from which the economy experiments (asymptotic) divergence exhibits pollution levels less than the irreversibility threshold \(\bar{P}\) once the quota is relatively important. In other words, choosing the strictest quota minimizes, indeed rules out, the risk, for an economy that does not initially suffer from the irreversibility of environmental damages, to follow a development trajectory characterized by the impoverishment in environmental and physical capitals.

\(^{17}\)The belonging to this region is a determining factor to explain the process of divergence.

\(^{18}\)The stable reversible solution is unique here and corresponds to the PMSS defined in proposition 2.
Figure 3: Basin of attraction and divergence region when \( E = 0.6 \)

Figure 4: Basin of attraction and divergence region when \( E = 1.2 \)
This property seems quite natural insofar as a high quota of permits contributes to reinforce the environmental constraint weight. The rhythm of pollution accumulation is more sustained and consequently the recovery process of nature is saturated faster. In turn, agents react by giving the priority to maintenance expenditures at the expense of wealth accumulation. The impoverishment mechanism described above will finally arise for lower pollution levels (and less than the irreversibility threshold).

From this numerical example, we then confirm the results obtained, as for PM solutions are concerned, during the stationary analysis since the observations tend to recommend to announce the lowest quota provided that it is greater than the critical threshold $\bar{E}_L$.

Before ending this discussion, we have to express the following remark. The fact to emit a quota $E = \bar{E}_L + \varepsilon$, with $\varepsilon > 0$ infinitesimal, protects the economy not only from a convergence toward an environmental and poverty trap but also from a process of divergence as soon as its environment, at the initial period, is safe.\(^\text{19}\) However, it remains the hypothetical case where initial pollution is already irreversible. Considering this extreme situation would logically induce us to review appreciably our conclusions. In this situation, one can expect that public authorities will have to set the quota to a very low level, and less than $\bar{E}_L$, in order to allow the economy to stabilize at a stationary trap. The convergence toward these states may finally constitute a lesser evil with regard to the perpetual impoverishment that goes with the asymptotic trap.

5 Conclusion

In an OLG model with irreversible pollution, Prieur [2007] has shown that a possible outcome of the development process, without pollution control, is the convergence toward an economic and ecological poverty trap. This paper first addresses the question to know whether or not, a pollution regulation through the implementation of a permits system is a means to prevent the economy from reaching a trap. In this framework, the economy can potentially face two traps of different nature. The first one is a steady state, with an irreversible level of pollution and a low level of wealth, in which the economy can stabilize in the long run. The second one is an asymptotic trap in the sense that it corresponds to a growth path associated with a perpetual erosion in both economic and environmental resources. The analysis reveals the existence of a critical threshold for polluting emissions. Now, choosing an emission quota above this level is a means to avoid the "stationary" trap. Moreover, fixing the quota at the lowest level beyond this threshold is also sufficient to protect the economy, that is not initially endowed with an irreversible level of pollution, against the recession going with the asymptotic trap.

In the context of the absence of traps, we next turn to the analysis of an environmental policy reform that consists of a reduction of the global quota on emissions. Its repercussions are widely

\(^{19}\)This is the case of study that \textit{a priori} makes the more sense. In fact, the idea that we get about the role of a system of pollution regulation is precisely to intervene before facing an irreparable situation.
dependent on the type of equilibrium considered. In fact, the equilibria with reversible pollution
are only distinguished by the fact that private agents engage, or not, in maintenance. Therefore,
at the zero maintenance solution, the fall in the quota effectively causes a decrease in the level of
stationary pollution. But, this effort is detrimental to capital accumulation. In other words, there
exists a dilemma between pollution control and economic growth. On the contrary, at the positive
maintenance equilibrium, we show that a tightening of environmental policy goes with both a
fall in pollution and a rise in capital at steady state. Thus, an environmentally ambitious reform
of the permits system brings a double dividend to the economy. This striking result echoes the
conclusions of the literature on tax reform (see notably Bovenberg and Smulders [1995], [1996] and
Bovenberg and de Mooij [1997]). However, in contrast with these papers, it does not depend on
the controversial assumption of the existence of a strong environmental externality in production.
References


A  Existence conditions for PME (Proposition 2)

A.1  Properties of $m(k,E)$

The function $m(k,E)$ is defined as:

$$m(k,E) = (1 - \alpha)A\bar{E}^{1-a-\beta} k^{\alpha} - k.$$ 

It represents the stationary maintenance effort in the model. The partial derivatives with respect to $k > 0$ and $\bar{E} > 0$ are:

$$
\begin{align*}
    m_1(k,\bar{E}) &= \alpha(1 - \alpha)Ak^{\alpha-1}\bar{E}^{1-a-\beta} - 1 \\
    m_{11}(k,\bar{E}) &= -\alpha(1-\alpha)^2 Ak^{\alpha-2}\bar{E}^{1-a-\beta} < 0 \\
    m_2(k,\bar{E}) &= (1 - \alpha - \beta)(1 - \alpha)Ak^\alpha \bar{E}^{a-\beta} > 0 \\
    m_{22}(k,\bar{E}) &= -(\alpha + \beta)(1 - \alpha - \beta)(1 - \alpha)Ak^\alpha \bar{E}^{a-\beta-1} < 0,
\end{align*}
$$

whereas the cross derivative writes:

$$m_{12}(k,\bar{E}) = \alpha(1-\alpha)(1-\alpha-\beta)Ak^{\alpha-2}\bar{E}^{1-a-\beta} > 0.$$ 

For a given $\bar{E} > 0$, we have therefore:

- There exists a unique $k \in [0, +\infty)$ such that $m(k,\bar{E}) = 0$, $\bar{k}(\bar{E})$, given by:

$$\bar{k}(\bar{E}) = \left[ (1 - \alpha)A\bar{E}^{1-a-\beta} \right]^{1/\alpha} ,$$

and $m(k,\bar{E}) \geq 0$ for $k \in [0, \bar{k}(\bar{E})]$. Moreover, we see that $\bar{k}'(\bar{E}) > 0$: the upper bound of the interval is increasing with respect to the quota. It is worth noting that this upper bound exactly corresponds to the level of capital $k^*_2(\bar{E})$ reached at the ZMSS (see Section 3.1).

- There exists a unique $k \in [0, \bar{k}(\bar{E})]$ such $m_1(k,\bar{E}) = 0$. Let $\tilde{k}(\bar{E})$ be this value:

$$\tilde{k}(\bar{E}) = \left[ \alpha(1 - \alpha)A\bar{E}^{1-a-\beta} \right]^{1/\alpha} .$$

We know that $m_1(k,\bar{E}) \geq 0$ for $k \in [0, \tilde{k}(\bar{E})]$. The maximal value reached by the function is:

$$m(\tilde{k}(\bar{E}),\bar{E}) = A(1 - \alpha)^2 \left[ \alpha(1 - \alpha)A \right]^{\alpha/(1-a)} \bar{E}^{1-a-\beta} ,$$

which is $> 0$ for all $\bar{E} > 0$. Note that $\tilde{k}'(\bar{E}) > 0$, for all $\bar{E} > 0$: the location of the maximum of $m$ is also increasing with respect to the quota.
A.2 Existence of an asymptotic trap

First, observe that as \( \varphi(k) \) is decreasing, so is \( \varphi^{-1}(P) \). Assume \( \bar{E} \) is fixed, and pick a value \( \eta \in (0, \bar{E}) \). Using a recurrence, we prove that if \( (k_t, P_t) \) is such that:

\[
P_t \geq \bar{P}, \quad P_t = \varphi(k_t) \quad \text{and} \quad \Gamma(P_t) + \gamma(1 - \alpha)\bar{E}^{1 - \alpha} k_t^\alpha \leq \bar{E} - \eta,
\]

for \( t = t_0 \), then for all \( t \geq t_0 \), Property (39) holds, and \( P_{t+1} \geq P_t + \eta \). This will imply that \( P_t \to \infty \), hence the asymptotic environmental trap. Since \( \lim_{k \to 0} \varphi(k) = +\infty \), we have: \( \lim_{P \to \infty} \varphi^{-1}(P) = 0 \). This will imply \( k_t \to 0 \), hence the asymptotic poverty trap.

Assume therefore that (39) holds. The dynamics (25) for \( P_t \) implies:

\[
P_{t+1} = P_t - \Gamma(P_t) + E - \gamma(1 - \alpha)\bar{E}^{1 - \alpha} k_t^\alpha + \gamma k_{t+1} \\
\geq P_t + \bar{E} - \left( \Gamma(P_t) + \gamma(1 - \alpha)\bar{E}^{1 - \alpha} k_t^\alpha \right) \\
\geq P_t + \eta.
\]

But since \( \varphi^{-1}(P) \) is decreasing, we have: \( k_{t+1} = \varphi^{-1}(P_{t+1}) \leq \varphi^{-1}(P_t) = k_t \). Using the fact that \( \Gamma(P) \) is also decreasing for \( \Gamma(P) \) for \( P \geq \bar{P} \), we finally have:

\[
\Gamma(P_{t+1}) + \gamma(1 - \alpha)\bar{E}^{1 - \alpha} k_{t+1}^\alpha \leq \Gamma(P_t) + \gamma(1 - \alpha)\bar{E}^{1 - \alpha} k_t^\alpha,
\]

and Property (39) holds at \( t + 1 \).

To conclude with the existence of an asymptotic poverty and environmental trap, observe that there indeed exist values \( (k_t, P_t) \) such that (39) holds. It is sufficient to pick \( k_t \) small enough so that \( \gamma(1 - \alpha)\bar{E}^{1 - \alpha} k_t^\alpha \leq \bar{E} - \eta \), and \( P_t = \max(\varphi(k_t), \bar{P}) \).

A.3 Existence of steady states with positive maintenance

According to equation (27), studying existence boils down to comparing the behaviour of two functions of \( k, \bar{E} \) being given. The general form of these two functions is displayed in figure 5.

The first function \( \theta(k) := \Theta(k, \bar{E}, k) = \bar{E} - \gamma m(k, \bar{E}) \) represents the amount of stationary emissions. Its behaviour derives from the properties of \( m(k, \bar{E}) \): \( \theta'(k) = -\gamma m_1(k, \bar{E}), \theta(0) = \theta(\bar{k}(\bar{E})) = \bar{E} \). It is first decreasing until \( \bar{k}(\bar{E}) \), then increasing until \( \bar{k}(\bar{E}) \). It is convex: \( \theta''(k) > 0 \). Thus, \( \theta(k) \) has a U shape.

The second function \( \Lambda(k) = \Gamma(\varphi(k)) \) is obtained by substituting the expression of \( P \) given by (26) in the assimilation function. The sign of its derivative, \( \Lambda'(k) = \varphi'(k)\Gamma'(\varphi(k)) \), follows from the properties of \( \Gamma() \) since \( \varphi'(k) < 0 \): it is negative when \( \varphi(k) \in [0, \bar{P}] \leftrightarrow k \in [\varphi^{-1}(\bar{P})], \varphi^{-1}(0)] \) while it is positive for any \( k \in [\varphi^{-1}(\bar{P})], \varphi^{-1}(0)] \). Thus, \( \Lambda(k) \) inherits from the inverted U shape of \( \Gamma() \). Moreover, we have \( \Lambda(\varphi^{-1}(\bar{P})) = 0 \) and \( \Lambda(\varphi^{-1}(\bar{P})) = \Gamma(\bar{P}) = \Gamma_{\max} \).

The main difficulty we encounter when comparing \( \theta(k) \) and \( \Lambda(k) \) is that the ranking between values of \( k \) determining the properties of \( \theta(k) \) (\( \bar{k}(\bar{E}), \bar{k}(\bar{E}) \)) and those related to the behaviour
of \( \Lambda(k) \) \((\varphi^{-1}(\tilde{P}), \varphi^{-1}(\tilde{P})\)...) is \textit{a priori} unknown. Thus, there is no other option but to impose technical conditions in order to ensure some correspondence between the domains of definition of these two functions.

In fact, if we assume \( \bar{k}(\bar{E}) > \varphi^{-1}(\tilde{P}) \) then two kinds of equilibria may exist. Their corresponding levels of capital belong to three different sub-intervals of the relevant domain of variation of \( k \), that is \([0, \bar{k}(\bar{E})]\).

\[ \rightarrow \text{Environmental traps:} \] A Steady state located on the interval \([0, \varphi^{-1}(\tilde{P})]\) is an \( ET \). Referring to figure 5, we see that such a SS exists if and only if the curves \( \Lambda(k) \) and \( \theta(k) \) intersect for some \( k < \varphi^{-1}(\tilde{P}) \), that is to say: that the curve \( \theta(k) \) crosses the horizontal axis. A necessary condition for this is that the minimal value of \( \theta(k) \) be negative. This minimal value is \( \bar{E} - \gamma m(\bar{k}(\bar{E}), \bar{E}) \) where the value of \( \bar{k} \) has been established in Eq.A.1. A direct computation reveals that the relation \( \gamma m(\bar{k}(\bar{E}), \bar{E}) \geq \bar{E} \) holds if and only if \( \bar{E} \leq \bar{E}_L \) with:

\[
\bar{E}_L = \left(\gamma A(1-\alpha)^2\right)^{\frac{1}{\gamma}} \left(\alpha(1-\alpha)A\right)^{\frac{1}{\gamma}}.
\]

Hence the necessary condition. This condition is not sufficient however, since it remains possible that the two curves cross each other for some \( k > \varphi^{-1}(\tilde{P}) \). If the curve \( \theta(k) \) crosses the horizontal axis, it can do it at most twice since it is convex.

According to the sufficient conditions for local stability/unstability (40) and (41) (see appendix B.2), the SS with the lowest capital is unstable (since it verifies \( \theta'(k) < 0 \) and \( \Lambda'(k) = 0 \)) whereas the second (with \( \theta'(k) > 0 \)) is locally stable. Thus, the latter is the poverty trap.
→ **Steady states with reversible pollution:** Concerning the existence of SS with reversible pollution, two cases have to be envisioned.

- On the interval \([\varphi^{-1}(\bar{P}), \varphi^{-1}(\hat{P})]\):
  - Assume first that \(\tilde{k}(\bar{E}) > \varphi^{-1}(\bar{P})\). If \(\bar{E} > \bar{E}_L\) and (19) holds that is, if \(\Gamma_{\text{max}} \geq \bar{E}\), then there exists a unique SS when the curves cross each other, \(\theta(k)\) being decreasing and \(\Lambda(k)\) being increasing. But this SS is unstable since it meets the sufficient condition for instability.
  
  It may also exist SS that correspond to intersections between the two curves when both are increasing. These SS alternatively satisfy \(\theta'(k) > \Lambda'(k)\) and the reverse, these inequalities respectively being the sufficient condition for local stability and instability.

- Consider now the case \(\tilde{k}(\bar{E}) < \varphi^{-1}(\bar{P})\): only the second type of equilibrium exists if \(\bar{E} > \bar{E}_L\) and (19) holds. Under these conditions, the curves cross each other at least once. Since \(\Lambda(\varphi^{-1}(\bar{P})) < \theta(\varphi^{-1}(\bar{P}))\) and \(\Lambda(\varphi^{-1}(\bar{P})) \geq \bar{E} > \theta(\varphi^{-1}(\bar{P}))\), then one intersection lies in the interval \([\varphi^{-1}(\bar{P}), \varphi^{-1}(\hat{P})]\). Note that it cannot be deduced, without further assumptions, that this intersection is unique since \(\Lambda(k)\) is not convex.

- On the interval \([\varphi^{-1}(\bar{P}), \tilde{k}(\bar{E})]\), there is a unique SS if \(\tilde{k}(\bar{E}) > \varphi^{-1}(\bar{P}_{cr})\) and (19) holds. The technical condition enables to rank the two functions at the upper bound \(\tilde{k}(\bar{E})\) since: \(\theta(\tilde{k}(\bar{E})) = \bar{E} > \Lambda(\tilde{k}(\bar{E}))\). In addition with (19), it is sufficient to ensure the existence of an intersection on the interval. Since it is more restrictive than \(\tilde{k}(\bar{E}) > \varphi^{-1}(\bar{P})\), it is the one that appears in proposition 2.

This steady state \((k^{*+}_{ir}, \bar{P}^{*+}_{ir})\), with \(\theta'(k) > 0\) and \(\Lambda'(k) < 0\), is locally stable (see appendix B.2). Among the PMSS with reversible pollution, it is the one associated with the highest wealth and, according to the equilibrium relation (24), the lowest pollutant concentration.

### B Local Dynamics and Stability

#### B.1 The ZMSS

In the dynamics system (16), the two variables are independent. The dynamics for \(\{k_t\}\) write as:

\[
k_{t+1} = \Omega(k_t) = m(k_t, \bar{E}) + k_t.
\]

It is possible to deduce from the analysis of the function \(m\) (see A.1) that at the fixed point \(\tilde{k}(\bar{E})\), \(0 < \Omega'(\tilde{k}(\bar{E})) < 1\). This implies the local stability for \(k^{*}_{cr}(\bar{E})\).

Next, consider the dynamics for \(P\). In the case \(\Gamma_{\text{max}} = \bar{E}\), there is a unique fixed point \(P^{*}_{cr}(\bar{E}) = \bar{P}\). In that case, the sequence \(\{P_t\}\) is increasing if \(P_0 \leq \bar{P}\), and it converges to \(\bar{P}\). However, if \(P_0 > \bar{P}\), the sequence diverges to \(+\infty\).
In the case $\Gamma_{\text{max}} > \bar{E}$, there are two fixed points located on each sides of $\bar{P}$: $P_{cr}^+ (\bar{E}) < \bar{P} < P_{cr}^+ (\bar{E})$. Moreover, from Assumption 1, $\Gamma'(P_{cr}^- (\bar{E})) < 0 < \Gamma'(P_{cr}^+ (\bar{E})) < 1$. On the other hand, by linearizing the system (16), we get:

$$dP_{t+1} = (1 - \Gamma'(P_{cr}^+ (\bar{E})))dP_t .$$

It follows that the SS $P_{cr}^- (\bar{E})$ is stable, but the SS $P_{cr}^+ (\bar{E})$ is unstable.

### B.2 The PMSS

For a reversible SS $(k_{ir}^0 (\bar{E}), P_{ir}^0 (\bar{E}))$, the linearization of (25) gives the Jacobian matrix ($J$):

$$
\begin{pmatrix}
\frac{dk_{t+1}}{dP_{t+1}} \\
\frac{dP_{t+1}}{dP_{t+1}}
\end{pmatrix}
= \frac{1}{\varphi'(k_{ir}^0 (\bar{E})) - \gamma}
\begin{pmatrix}
-\gamma \Omega_1 (k_{ir}^0 (\bar{E}), \bar{E}) & 1 - \Gamma'(P_{ir}^0 (\bar{E})) \\
-\gamma \Omega_1 (k_{ir}^0 (\bar{E}), \bar{E}) \varphi'(k_{ir}^0 (\bar{E})) & (1 - \Gamma'(P_{ir}^0 (\bar{E}))) \varphi'(k_{ir}^0 (\bar{E}))
\end{pmatrix}
\begin{pmatrix}
dk_t \\
dP_t
\end{pmatrix} .
$$

Now, it is clear that $\det(J) = 0$. In fact, due to the equilibrium relation (24), the system reduces to a one-dimensional dynamics. Thus, the two eigenvalues are 0 and $\text{tra}(J)$:

$$\text{tra}(J) = \frac{-\gamma \Omega_1 (k_{ir}^0 (\bar{E}), \bar{E}) + (1 - \Gamma'(P_{ir}^0 (\bar{E}))) \varphi'(k_{ir}^0 (\bar{E}))}{\varphi'(k_{ir}^0 (\bar{E})) - \gamma} .$$

According to Assumption 1, $\Gamma'(P) < 1 \forall P$. Following the same reasoning as in Section B.1, we have $\Omega_1 () > 0$. Since $\varphi'() < 0$, the denominator is negative, and the expression above is positive. Therefore, there will be local stability if $\text{tra}(J) < 1$ and unstability if $\text{tra}(J) > 1$.

The following condition is sufficient for local stability:

$$\gamma (1 - \Omega_1 (k_{ir}^0 (\bar{E}), \bar{E})) > \Gamma'(P_{ir}^0 (\bar{E})) \varphi'(k_{ir}^0 (\bar{E})) .$$

It is equivalent to: $\theta'(k_{ir}^0 (\bar{E})) > \Lambda'(k_{ir}^0 (\bar{E}))$.

Under condition (29) in Proposition 2, the equilibrium $k_{ir}^{+ -}$ satisfies:

$\bar{k} (\bar{E}) < k_{ir}^{+ -} < \bar{k} (\bar{E})$

and,

$P_{ir}^{+ -} (\bar{E}) < P_{cr}^{+ -} (\bar{E}) < \bar{P}$

thus, it is such that: $\Omega_1 (k_{ir}^{+ -} (\bar{E}), \bar{E}) < 1$ and $\Gamma'(P_{ir}^{+ -} (\bar{E})) > 0$, and the inequality (40) holds. The equilibrium is therefore stable.

Finally, the following condition is sufficient for local unstability:

$$\gamma (1 - \Omega_1 (k_{ir}^0 (\bar{E}), \bar{E})) < \Gamma'(P_{ir}^0 (\bar{E})) \varphi'(k_{ir}^0 (\bar{E})) .$$

$\Theta'(k_{ir}^0 (\bar{E})) < \Lambda'(k_{ir}^0 (\bar{E}))$.

Any PME with $k_{ir}^0$ (if it exists), satisfying $\Omega_1 (k_{ir}^0 (\bar{E}), \bar{E}) > 1$ and $\Gamma'(P_{ir}^0 (\bar{E})) < 0$, is therefore unstable.
C Proof of proposition 5

In appendix A.3, we have analyzed the existence of an PM reversible SS by comparing the behaviour of two functions of $k$ for a given quota $\bar{E}$. Now, we consider the impact of a change in $\bar{E}$ on the equilibrium outcome $(k^*_{ir}(\bar{E}), P^*_{ir}(\bar{E}))$.

The steady state solves the following system:

\[
\begin{cases}
P^*_{ir} = \Phi(k^*_{ir}, \bar{E}) \\
\Gamma(P^*_{ir}) = \Theta(k^*_{ir}, \bar{E}, k^*_{ir})
\end{cases}
\]

By substituting the equilibrium relation in the second equation, we get:
\[
\Gamma(\Phi(k^*_{ir}, \bar{E})) = \Theta(k^*_{ir}, \bar{E}, k^*_{ir}).
\]

This equation implicitly defines $k^*_{ir}$ as a function of $\bar{E}$: $k^*_{ir} = k^*_{ir}(\bar{E})$ with,
\[
\frac{dk^*_{ir}}{d\bar{E}} = \frac{\Theta_2 - \Phi_2 \Gamma'}{\Phi_1 \Gamma' - \Theta_k}.
\]

With a slight abuse of notation, we have denoted:
\[
\Theta_k = \frac{d}{dk}\Theta(k, \bar{E}, k).
\]

Our analysis only makes sense for the stable SS, hence we refer to the two sufficient conditions for local stability: $\Omega_1(k^*_{ir}(\bar{E}), \bar{E}) < 1$ and $\Gamma'(P^*_{ir}(\bar{E})) > 0$. The sign of the partial derivatives $\Phi_1$ and $\Phi_2$ being known, it remains to determine the sign of $\Theta_2$. The emissions function writes $\Theta(k, \bar{E}, k) = \bar{E} - \gamma m(k, \bar{E})$. For any $k$, its derivative with respect to $\bar{E}$ is: $\Theta_2(k, \bar{E}, k) = 1 - \gamma \Omega_2(k, \bar{E})$. Since $\Omega_2(k, \bar{E}) > 0$, $\Omega_2$ is increasing in $k$. Computing its value at the upper bound $\tilde{k}(\bar{E})$ yields:
\[
\Omega_2(\tilde{k}(\bar{E}), \bar{E}) = (1 - \alpha - \beta)(A(1 - \alpha))^{1/(1-\alpha)} \bar{E}^{-\beta/(1-\alpha)}.
\]

Now, it appears that $\Omega_2(\tilde{k}(\bar{E}), \bar{E}) \leq 1/\gamma$ is equivalent to:
\[
\bar{E} \geq \bar{E}_c := (\gamma(1 - \alpha - \beta))^{(1-\alpha)/\beta}(A(1 - \alpha))^{1/\beta}.
\]

For any $\bar{E} \geq \bar{E}_c$, we have $\Theta_k(k, \bar{E}, k) \geq 0 \ \forall k \in (\varphi^{-1}(\tilde{P}), \tilde{k}(\bar{E})).$\(^{20}\) Therefore, if this inequality holds (condition (34) in Prop. 5), then it appears that $k^*_{ir}(\bar{E}) < 0$.

\(^{20}\)Since we restrict the analysis to quotas that are greater than the threshold $\tilde{E}_L$, imposing $\tilde{E}_L \geq \bar{E}_c$ is sufficient to conclude. More precisely, $\tilde{E}_L \geq \bar{E}_c$ holds when and only when
\[
\beta \geq (1 - \alpha)(1 - \alpha^\alpha/(1-\alpha)) \ .
\]

This bound is not very restrictive. If we suppose that the share of labour in production $1 - \nu$ belongs the range $[0.6, 0.7]$ (which is the common range for the estimations of this parameter), then this inequality is satisfied, for instance, for $\zeta = 1$. 

36
Next, we replace $k^*_{ir}$ with $k^*_i(E)$ in the equilibrium relation so as to compute the derivative of $P^*_{ir}(E)$. We get

$$P''_{ir}(E) = \frac{\Phi_1 \Theta_2 - \Phi_2 \Theta_k}{\Phi_1 \Gamma' - \Theta_k},$$

and this expression is equivalent to:

$$P''_{ir}(E) = \frac{U_1(\Theta_k R_2 - \Theta_2 R_1) + (RU_{11} + \gamma U_{12})(\Theta_k C_2 - \Theta_2 C_1)}{(\Phi_1 \Gamma' - \Theta_k)(RU_{12} + \gamma U_{22})}.$$  

The denominator and the first term in the numerator are positive. Since $RU_{11} + \gamma U_{12} < 0$, imposing $\Theta_k c_2 - \Theta_2 c_1 < 0$ (second condition in Prop. 5) ensures $P''_{ir}(E) > 0$. This condition rewrites:

$$\frac{\Theta_k(k^*_i(E), E, k^*_i(E))}{\Theta_2(k^*_i(E), E, k^*_i(E))} < \frac{c_1(k^*_i(E), E)}{c_2(k^*_i(E), E)}.$$  

Now, note that consumption and emissions, at steady state, express as follows:

$$c(k^*_i(E), E) = R(k^*_i(E), E) = c^*_i(E)$$

$$\Theta(k^*_i(E), E, k^*_i(E)) = E - \gamma m(k^*_i(E), E) = \Theta^*_i(E)$$

and their derivative with respect to $E$ reads respectively:

$$c''_{ir}(E) = c_1 k''_{ir}(E) + c_2$$

$$\Theta''_{ir}(E) = \Theta_k k''_{ir}(E) + \Theta_2.$$  

Thus imposing $c''_{ir}(E) < 0$ and $\Theta''_{ir}(E) > 0$ implies:

$$-\frac{\Theta_k(k^*_i(E), E, k^*_i(E))k''_{ir}(E)}{\Theta_2(k^*_i(E), E, k^*_i(E))} < 1 < -\frac{c_1(k^*_i(E), E)k''_{ir}(E)}{c_2(k^*_i(E), E)}$$

and the condition (35) follows from this ranking.

D Admissibility Analysis

D.1 The indifference Frontier

First of all, under our specifications for $U(c, P)$ and $\Gamma(P)$, note that the FOC (and the equilibrium relation) reads:

$$P_t = \frac{1}{\gamma \phi k_t}.$$  

The expression of the frontier $f(k_t, E)$ then follows from the FOC in which we set $m_t = \mu = 0$:

$$-\frac{1}{\Omega(k_t, E)} + \gamma \phi (P_t - \Gamma(P_t) + \bar{E}) = 0.$$  

(42)

Its particular shape is dependent on the level of pollution (reversible or not):
- when pollution is irreversible, the frontier is given by:

\[
P_t = f^1(k_t, \bar{E}) = \frac{1}{\gamma \phi (1 - \alpha) A k_t^\alpha E^{1 - \alpha - \beta}} - \bar{E}
\]  

(43)

and this expression is valid as long as \( P_t \geq \bar{P} \), or:

\[
k_t \leq \hat{k}(\bar{E}) = \left( \frac{1}{\gamma \phi (P + \bar{E})(1 - \alpha) A E^{1 - \alpha - \beta}} \right)^{\frac{1}{\alpha}}.
\]

If \( \hat{k}(\bar{E}) > \hat{k}(\bar{E}) \), then it is always located above the irreversibility threshold.

Assume now that \( \hat{k}(\bar{E}) < \hat{k}(\bar{E}) \),\(^{21}\) for any \( k_t > \hat{k}(\bar{E}) \), the computation of the frontier requires to solve the following polynomial equation:

\[
P_t (1 - \theta (\bar{P} - P_t)) - \left( \frac{1}{\gamma \phi (1 - \alpha) A k_t^\alpha E^{1 - \alpha - \beta}} - \bar{E} \right) = 0.
\]

Let us suppose that the discriminant is positive,

\[
\Delta^f = (1 - \theta \bar{P})^2 + 4\theta \left( \frac{1}{\gamma \phi (1 - \alpha) A k_t^\alpha E^{1 - \alpha - \beta}} - \bar{E} \right) > 0.
\]

Direct calculations reveal that the first root is always negative and can be excluded. Thus, we focus on the second root that is \textit{a priori} associated with positive and lower than the threshold \( \bar{P} \) values of pollution

\[
P_t = \frac{-(1 - \theta \bar{P}) + \sqrt{\Delta^f}}{2\theta}. \tag{44}
\]

The function in (44) is monotonic decreasing in \( k \). So, there exists a unique value \( \hat{k}(\bar{E}) \) such that it crosses the abscissa \( (P = 0) \) with,

\[
\hat{k}(\bar{E}) = \left( \frac{1}{\gamma \phi \bar{E}(1 - \alpha) A E^{1 - \alpha - \beta}} \right)^{\frac{1}{\alpha}}
\]

and it appears that, for any \( k_t \leq \hat{k}(\bar{E}) \), the square root in (44) is non-negative. Moreover, we confirm that \( \Delta^f \) is strictly positive on \([0, \hat{k}(\bar{E})]\).

To summarize, since we only consider non-negative levels of pollution, the expression of the frontier, when pollution is reversible, is:

\[
P_t = f^2(k_t, \bar{E}) = \begin{cases} 
- \frac{(1 - \theta \bar{P}) + \sqrt{\Delta^f}}{2\theta} & \text{for } k_t \leq \hat{k}(\bar{E}) \\
0 & \text{otherwise.}
\end{cases} \tag{45}
\]

Beyond the level \( \hat{k}(\bar{E}) \), it precisely meets the abscissa. It means that the agents engage in maintenance regardless of the level of pollution.

\(^{21}\)This condition boils down to impose, for instance,

\[
A \geq \frac{1}{(\gamma \phi (P + \bar{E}))^{1 - \alpha} (1 - \alpha) E^{1 - \alpha - \beta}}
\]
D.2 Admissibility of reversible SS

For a ZM solution, it is straightforward to see that \( P_{cr}^*(\bar{E}) < \bar{P} \). The admissibility also requires \( P_{cr}^*(\bar{E}) < f(k_{cr}^*(E), \bar{E}) \). If \( k_{cr}^*(\bar{E}) = \check{k}(\bar{E}) \), then the frontier is always located above the threshold \( \bar{P} \), on the range \([0, \check{k}(\bar{E})]\). Thus the solution is admissible since \( P_{cr}^*(\bar{E}) < \bar{P} \). Otherwise \( (\check{k}(\bar{E}) > \check{k}(ar{E})) \), two cases possibly arise:

- if \( \check{k}(\bar{E}) \leq \check{k}(\bar{E}) \), we have to set a condition on parameters to ensure \( P_{cr}^*(\bar{E}) < f(k_{cr}^*(E), \bar{E}) \).
- if \( \check{k}(\bar{E}) > \check{k}(\bar{E}) \), then the solution is inadmissible because \( P_{cr}^*(\bar{E}) > f(k_{cr}^*(E), \bar{E}) = 0 \).

For the positive maintenance SS, by construction we have \( k_{ir}^*(\bar{E}) > \varphi^{-1}(\bar{P}) \leftrightarrow P_{ir}^*(\bar{E}) < \bar{P} \). It remains to study its location with respect to the indifference frontier. Assume first that \( k_{ir}^*(\bar{E}) \leq \check{k}(\bar{E}) \). On the range \([0, \check{k}(\bar{E})]\), the frontier is above \( \bar{P} \), so we have \( f(k_{ir}^*(\bar{E}), \bar{E}) > \bar{P} > P_{ir}^*(\bar{E}) \): the solution is not admissible since it is finally located in the constraint region. Assume next that \( k_{ir}^*(\bar{E}) > \check{k}(\bar{E}) \), in this case, it is always admissible since:

- if \( k_{ir}^*(\bar{E}) \in (\check{k}(\bar{E}), \check{k}(\bar{E})) \), then direct calculations show that \( P_{ir}^*(\bar{E}) > f(k_{ir}^*(\bar{E}), \bar{E}) \iff \gamma m(k_{ir}^*(\bar{E}), \bar{E}) > 0 \) and this inequality is satisfied.
- if \( k_{ir}^*(\bar{E}) > \check{k}(\bar{E}) \), we necessarily have \( P_{ir}^*(\bar{E}) > f(k_{ir}^*(\bar{E}), \bar{E}) = 0 \).

To summarize, the following double inequality: \( \bar{k}(\bar{E}) < \check{k}(\bar{E}) \leq \check{k}(\bar{E}) \) is a necessary condition under which the two types of solutions are simultaneously admissible. In fact, if \( \check{k}(\bar{E}) \leq \check{k}(\bar{E}) \) then the constraint solution alone is admissible while if, on the contrary, \( \check{k}(\bar{E}) > \check{k}(\bar{E}) \), only the interior SS is potentially admissible.

The condition \( \check{k}(\bar{E}) \leq \check{k}(\bar{E}) \), for a constraint SS, can be rewritten as follows: \( \bar{E} \leq \bar{E}_a \) with

\[
\bar{E}_a = \left( \frac{1}{A(1-\alpha)(\gamma\phi)^{1-\alpha}} \right)^{1/(2(1-\alpha)-\beta)}.
\]

The inequality \( \check{k}(\bar{E}) < \check{k}(\bar{E}) \) is equivalent to:

\[
((A(1-\alpha)\bar{E}^{1-\alpha-\beta})^{1/(1-\alpha)} \gamma\phi(\bar{E} + \bar{P}) > 1.
\]

We cannot express it in terms of a condition on \( \bar{E} \). But, if we note that \( \bar{E} < \bar{P} \), the right-hand side is greater than

\[
2(A(1-\alpha)\bar{E}^{1-\alpha-\beta})^{1/(1-\alpha)} \gamma\phi\bar{E}.
\]

Imposing that this expression is greater than one, which is equivalent to

\[
\bar{E} \geq \bar{E}_i = \left( \frac{1}{2A(1-\alpha)(\gamma\phi)^{1-\alpha}} \right)^{1/(2(1-\alpha)-\beta)}
\]

is sufficient to have \( \check{k}(\bar{E}) \leq \check{k}(\bar{E}) \).
D.3 Basin of attraction and divergence

The dynamics in the region of active maintenance is given by the system of equations (25). Accordingly, every initial state of the system \((k_0, P_0)\) is mapped to the curve of equation \(P = \varphi(k)\). The set of initial positions that are mapped to a particular point \((k_1, P_1) = (k, P)\) of this curve is obtained by solving the second equation of (25) (written with \(t = 0\)) for \((k_0, P_0)\). This gives:

\[
P = P_0 - \Gamma(P_0) + \Theta(k_0, E, k)
\]

that is:

\[
k_0 = \left(\frac{P_0 - \Gamma(P_0) - P + E + \gamma k}{\gamma(1 - \alpha)AE^{1-a-\beta}}\right)^{\frac{1}{\beta}}.
\]

According to the dynamics (25), the state then stays on the curve of equation \(P = \varphi(k)\). Under appropriate conditions, there are two stable states. The initial positions corresponding to these two equilibria form two curves that delimitate three zones, as illustrated in figure 6. The zone at the top is the divergence zone. A trajectory that originate there goes to an asymptotic trap, as illustrated in figure 2. In the center zone, trajectories approach the stable equilibrium from above: the sequence \(k_t\) is increasing and the sequence \(P_t\) is decreasing, except possibly for the first step. In the bottom zone, trajectories approach the stable equilibrium from below.

![Diagram](image)

Figure 6: Convergence patterns. Left: without admissible constraint equilibrium. Right: without admissible interior equilibrium
E  Global Dynamics

In the constraint region, dynamics are given by:

\[ k_{t+1} = (1 - \alpha)A k_t^\alpha \bar{E}^{1-\alpha-\beta} \]

\[ P_{t+1} = P_t(1 - \theta(\bar{P} - P_t)) + \bar{E} \]

if pollution is reversible. Otherwise, the dynamics for pollution write:

\[ P_{t+1} = P_t + \bar{E}. \]

Dynamics, when maintenance is positive, become:

\[ k_{t+1} = \frac{1}{\gamma \phi P_{t+1}} \]

\[ P_{t+1} = \frac{x(k_t, P_t) + \sqrt{x(k_t, P_t)^2 + 4/\phi}}{2} \geq 0 \]

with,

\[ x(k_t, P_t) = P_t + \bar{E} - \gamma(1 - \alpha)A k_t^\alpha \bar{E}^{1-\alpha-\beta} \]

when pollution is irreversible and, if not:

\[ x(k_t, P_t) = P_t(1 - \theta(\bar{P} - P_t)) + \bar{E} - \gamma(1 - \alpha)A k_t^\alpha \bar{E}^{1-\alpha-\beta}. \]
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Stéphane MUSSARD : mussard@lameta.univ-montp1.fr