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Julie BEUGNOT

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Abstract

We introduce the heterogeneity of labor in a simple imperfectly competitive aggregate labor market model "à la Manning (1990)" in order to analyze the effects of an exogenous rise of the legal minimum wage on the unemployment equilibrium, the wage dispersion and the general price level. We assume also the presence of "knowledge spillovers" in the individual production function leading to increasing returns to scale at the aggregate level and involving the possibility of multiple equilibria. Then, thanks to a comparative statics exercise, we show that a rise in the legal minimum wage has no impact on the unemployment equilibria, increases the general price level proportionally to the share of low-skilled employment in the total employment and reduces the wage dispersion. These results are broadly consistent with the Card Krueger’s empirical findings (1995).

Keywords: multiple unemployment equilibria, minimum wage, general price level.

JEL Classification: D43, E24, J24, J31.
Introduction

The problem of minimum wage hikes has been actively discussed among economists. The primary goal of such a government’s intervention is to improve the welfare of low paid workers. However, many are those who think that an increase in the minimum wage leads also to employment losses for the workers at the bottom of the wage distribution as young workers or low-skilled workers. In their book, Card and Krueger (1995) argue that the data on the fast-food industry in U.S. states give no support to this idea. Machin and Manning (1994) analyze also the effect of the minimum wage cuts on employment in the U.K. and reach to the same conclusions. With these works, the debate has been revived as well in the academic area as in the political area.

The effect of minimum wage on employment has been analyzed especially with models like the efficiency wage model (Rebitzer and Taylor, 1995) or the monopsony model (Card and Krueger, 1995), i.e. models where the firm has a high power on wage determination. In the monopsony model, the firm consents to pay higher wages in order to reduce quit rates and make easier recruitments, while in the efficiency wage model higher wages intend to resolve the worker incentive problem. As a result, in such models a minimum wage can have a positive or null effect on employment. However, as mentioned by Cahuc et al. (2001), only few papers rely on models using wage bargaining with trade union; even though this way of wage determination appears as an important feature of European labor markets.

Furthermore, the sharp variations in European unemployment rates have induced economists to explain this fact by the movement from one equilibrium to another. Thus, the possibility of multiple equilibria in an economy has been highlighted. In his researches, Manning (1990, 1992) has developed this view and estimated a model for British data for the period 1951-1987. Theoretically, the possibility of multiple equilibria comes from the presence of strategic complementarities between agents. In his book, Cooper (1999) reports and analyzes in detail levels where they can occur: external or internal returns to scale in production technology, imperfect competition, and the way in which agents come together to trade (search and matching) and take their decisions (timing of choices).

According to these reports, the purpose of our paper is to examine the effects of minimum wage increase on labor market’s performance in a multiple equilibria model where the wage negotiation takes place between a firm and a trade union. The model used in the paper is essentially the imperfectly competitive model of Manning (1990) in which we introduce the heterogeneity of labor and the presence of knowledge spillovers in the individual production technology.

Considering heterogeneous workers is required when we want to focus the analysis on the effects of a minimum wage. Therefore, we consider that the labor force employed by the firm consists of low-skilled workers who are paid at the minimum wage and high-skilled workers who are paid at a negotiated wage. This negotiated wage results from a bilateral bargaining between the firm and the trade union which is assumed to be not dominated by skilled workers.
The assumption of knowledge spillovers is a convenient way to produce increasing returns to scale at the aggregate level while having constant returns to scale at the firm level. In the Manning’s model, the possibility of multiple equilibria is due to the presence of increasing returns to scale at the firm level, although this assumption has no strong theoretical justification. However, for several years, knowledge has been considered as a fundamental source of increasing returns to scale and a determinant of the persistence of productivity and income differentials across economic agents of production (Romer, 1986). Thus, in our model, the presence of knowledge spillovers allows us to give theoretical support for the presence of increasing returns to scale. These knowledge spillovers work through the average level of high-skilled labor employed in the economy, the high-skilled labor being both the engine and the carrier of knowledge.

The paper is organized as follows. In the first section, we present the model (the price behavior of the firm and the wage determination) and the general symmetric equilibrium. Secondly, we deal with the multiple equilibria existence issue. In this goal, we highlight the conditions under which this case occurs and we verify the existence of them. In a third section, we run a comparative statics analysis and we show that a minimum wage increase has no effect on the unemployment equilibria, increases general price level and reduces wage dispersion. Finally, we compare our theoretical findings with those of Card and Krueger (1995).

1 The model

We use the Manning’s model (1990) broadly based on the Layard-Nickell’s model (1985, 1986), in which we introduce the heterogeneity of labor and the presence of knowledge spillovers. In this model, the firms set their price, produce output and fix employment on the basis of the demand for their output and taking the wages as given. Then, they bargain with trade unions about the level of real wages.

1.1 The price behavior of the firm and its demand of labor

We assume monopolistic competition on the good market. The economy is made up of \( F \) identical imperfectly competitive firms, each producing an imperfectly substitutable good with others. Firm \( i \) has a production function of the form\(^1\):

\[
y_i = A n_{1i}^\alpha n_{2i}^\beta \quad \text{with} \quad \alpha + \beta = 1
\]

where \( n_{1i} \) represents its employment of high-skilled labor, \( n_{2i} \) its employment of low-skilled labor and \( A \) an efficiency parameter of labor. Since \( \alpha + \beta = 1 \), we have constant returns to scale at the firm level. \( A \) is assumed to be a function

\(^1\)As in the paper of Manning (1990), the number of firm is assumed fixed and the capital is excluded for simplicity.
of the average employment level of high-skilled labor in the economy $n_1$ and to have the following form:

$$A = n_1^\alpha$$

(2)

The knowledge spillovers present in the economy influence the firm’s production efficiency through this parameter $A$, where $n_1 = \frac{1}{F} \sum_{i=1}^{F} a_{1i}, \alpha \sigma > 0$ represents the size of these knowledge spillovers and $\sigma > 0$ is a measure of the degree of externalities. Indeed, given the fact that knowledge is the cause of new technologies development and mainly produced and spread by high-skilled labor force, considering that the production efficiency depends on the average use of high-skilled labor in an economy is a relevant assumption. Thus, the more the economy employs high-skilled labor, the more the labor employed by one firm and so its production is efficient. However, the firm is assumed to be too small to have any influence on the aggregate state of the economy, and so the firm $i$ takes this parameter $A$ as given during its optimization program. Thus, the positive externality of high-skilled employment is not internalised by the firm and it takes an advantage of high-skilled employment decisions of all other firms without having to pay for it.

Consequently, on account of the monopolistic competition and the presence of knowledge spillovers, we have two sources of strategic complementarities in this economy. Indeed, the production decision of each firm has a positive effect on the other firms’ production level, through the aggregate demand channel, as well as on the other firms’ production efficiency, through the knowledge spillovers channel. These strategic complementarities make possible the existence of multiple equilibria.

The total employment of firm $i$ writes:

$$n_i = n_{1i} + n_{2i}$$

(3)

Let $\gamma_i = n_{2i}/n_i \in ]0; 1[$ the proportion of low-skilled labor employment in the total employment for the firm $i$ and $\theta_i = w/w_{1i} \in ]0; 1[$ a measure of the wage dispersion between the high-skilled and low-skilled labor where $w$ represents the minimum wage which is earned by low-skilled workers and $w_{1i}$ the wage of high-skilled workers which is negotiated. The lower $\theta_i$ is, the larger the gap between the low-skilled worker’s wage and high-skilled worker’s wage is. Consequently, the total wage cost of the firm $i$ can be written as a function of the high-skilled worker’s wage $w_{1i}$, the total labor demand and the two parameters defined above:

$$w_i n_i = w_{1i} n_{1i} + \theta_i w_{1i} \gamma_i = [1 + (\theta_i - 1) \gamma_i] w_{1i} n_i = C(\gamma_i, \theta_i) w_{1i} n_i$$

(4)

with $C(\gamma_i, \theta_i) \in ]0; 1[$ and where $w_i = C(\gamma_i, \theta_i) w_{1i}$ represents a wage index paid by the firm to its workers. We can also write the production technology of the firm $i$ as a function of $n_i$ and $\gamma_i$:

$$y_i = B(\gamma_i) n_i$$

where

$$B(\gamma_i) = A(1 - \gamma_i)^\alpha \gamma_i^\beta$$

(5)
The demand for the firm i’s output is assumed to be given by:

\[ y_d^i = (Y/F) (p_i/P)^{-s}, \quad s > 1 \]  

(6)

where Y is the total aggregate demand, \( p_i \) the firm i’s price, \( P \) the general price level and \( s \) the demand elasticity in the good produced by the firm i.2

The real profit of the firm i can be written as:

\[ \pi_i/P = (p_i/P) y_i - C(\gamma_i, \theta_i)(w_{1i}/P)n_i \]  

(7)

Each firm i chooses \( p_i \) in order to maximize its real profit. When optimizing, the firm takes the aggregate state of the economy and the real wage of high-skilled workers as given, and produces exactly the amount addressed to it. Thus, it solves the following program:

\[ (p_i/P)^* = \arg \max_{p_i/P} \pi_i/P \]

s.t

\[ y_i = B(\gamma_i)n_i \]
\[ y_i = y_d^i = (Y/F) (p_i/P)^{-s} \]
\[ w_{1i}/P \quad \text{given} \]

The first order condition of this program gives us the following partial equilibrium pricing equation:

\[ (p_i/P)^* = \left( \frac{s}{s-1} \right) \frac{C(\gamma_i, \theta_i)}{AB(\gamma_i)} \left( \frac{w_{1i}}{P} \right) \]  

(8)

This equation shows that the price fixed by the firm is an increasing function of labor cost. Then, we use the equations (5), (6) and (8) to obtain an employment equation corresponding to the total labor demand of the firm i:

\[ n_i^* = \left[ \left( \frac{s}{s-1} \right) C(\gamma_i, \theta_i) \right]^{-s} [B(\gamma_i)]^{s-1} (Y/F) \left( \frac{w_{1i}}{P} \right)^{-s} \]  

(9)

The total labor demand of the firm has the common features found in the literature, i.e. it is downward sloping in the high-skilled real wage-employment space, decreasing in the labor cost3 and increasing in the total aggregate demand.

1.2 The wage determination at the firm level

The bargaining is decentralized and rests on the wage only, so we assume a "right-to-manage" model. The low-skilled labor wage corresponds to the legal minimum wage. It is fixed by law and not negotiated between the firm and the trade union. The bargaining concerns just the high-skilled labor wage \( w_{1i} \).

\( ^2 \)This specification of the demand function is derived from CES specification of preferences (see Blanchard and Kiyotaki (1987), and Julien and Sanz (2007)).

\( ^3 \)Indeed, we have \( \frac{\delta n_i^*}{\delta (w_{1i}/P)} = -s \left( \frac{1 - \gamma_i}{C(\gamma_i, \theta_i)} \right) n_i^* < 0. \)
The trade union hasn’t insider behavior and is not dominated by high-skilled workers. It cares about the welfare of both high-skilled and low-skilled employees for which the wage is fixed exogenously. We assume further that it weights equally the welfare of all employees of the firm and considers it as a whole during the negotiation. We assume also that it gives as much importance to the employment level as to the wage level of workers\(^4\). Its utility function is given by:

\[
V\left(\frac{w_i}{P}, n_i\right) = \left[\frac{w_i}{P} - w_R\right] n_i = \left[C(\gamma_i, \theta_i) \left(\frac{w_{1i}}{P}\right) - w_R\right] n_i
\]

where \(w_R\) is the real reservation wage which is exogenous at the firm level. This reservation wage represents the alternative utility of workers when the bargaining leads to none agreement between the firm and the trade union. Its specification will be given later. The firm’s utility is represented by its real profits.

The high-skilled real wage is negotiated in order to solve the following Nash bargaining program:

\[
\left(\frac{w_{1i}}{P}\right)^* = \arg\max_{\frac{w_{1i}}{P}} \left[\left(\frac{w_{1i}}{P}\right)^{(1-\rho)} \left\{C(\gamma_i, \theta_i) \left(\frac{w_{1i}}{P}\right) - w_R\right\} n_i\right]^\rho
\]

s.t \quad n_i = n_i^* \quad \text{given} \quad w_R

Where \(\rho \in [0; 1]\) represents the bargaining power of the union. The first order condition of this program gives us the high-skilled real wage that results from this negotiation:

\[
\left(\frac{w_{1i}}{P}\right)^* = \frac{\chi(\rho, s)}{C(\gamma_i, \theta_i)} w_R \quad \text{with} \quad \chi(\rho, s) = \left(\frac{\rho}{s - 1}\right) + 1 \geq 1
\]

Where \(\chi(\rho, s)\) is the mark-up. This wage equation is traditional and says that the high-skilled real wage is marked-up over the reservation wage with a mark-up which is greater than one and increasing in the bargaining power of union. Now, we turn to the general equilibrium of this economy.

1.3 The general equilibrium

At the general symmetric equilibrium, all the firms and trade unions are identical and take the same decisions. Thus, we have these following equalities:

\[p_i = P, w_{11} = w_1, n_{11} = n_1, n_{22} = n_2, n_i = n, y_i = y = Y/F, \theta_i = \theta \quad \text{and} \quad \gamma_i = \gamma\]

Furthermore, given the fact that all agents have taken their decisions at this step, we consider that the share of low-skilled labor in total employment \(\gamma\) and the aggregate wage dispersion \(\theta\) are constant in what follows.

Since the number of firms in the economy is assumed fixed, the aggregate production can be deduced from the sum of the \(F\) identical individual production technologies:

\(^4\)Here, the union’s preferences correspond to the utilitarian model of Oswald (1982).
At the aggregate level, we have increasing returns to scale (IRS) of labor due to the presence of knowledge spillovers in the economy and we note that the extent of these IRS depends on the knowledge spillovers size. Furthermore, these IRS are completely external to the firm but internal to the economy. They correspond to social increasing returns. Thus, the assumption of knowledge spillovers allows us to have IRS at the aggregate level while having constant returns to scale at the firm level and, in this way, reinforce the strategic complementarities already introduced by monopolistic competition.

Given the definition of the unemployment rate \( u = 1 - (Fn/N) \), where \( N \) represents the total labor force in the economy, we insert (12) in (9) and we obtain the aggregate price equation (PS for price setting) which relates the aggregate real wage of high-skilled labor to the unemployment rate \( u \) and other variables:

\[
\left( \frac{w_1}{P} \right)_{PS} = \left[ (1 - u) \frac{N/F}{s} \right]^\alpha \sigma \frac{\gamma^\sigma}{C(\gamma, \theta)} \frac{s - 1}{s - 1 - \frac{(1 - \gamma)^{1+\sigma}}{\gamma^\beta}}
\]

In order to determine the aggregate wage equation, we need to model the reservation wage which was exogenous at the firm level. Like Manning (1990), we use this convenient specification:

\[
w_R = u(B/P) + (1 - u)(w/P) = u(B/P) + (1 - u)C(\gamma, \theta)(w_1/P)
\]

Where \( u \) is the unemployment rate and \( B/P \) the real unemployment benefits and \( w/P \) the real wage index paid by the firms to the workers. The unemployment benefits are assumed to be the same for all the workers whatever their skill level and lower than the minimum wage. Thus, the reservation wage is a weighted average according to the unemployment rate of the workers’ earnings from employment and unemployment. We introduce (14) in (11) and we obtain the following aggregate wage equation (WS for wage setting) which relates also the real high-skilled labor wage to the unemployment rate and exogenous variables as the real unemployment benefits:

\[
\left( \frac{w_1}{P} \right)_{WS} = \frac{\chi(\rho, s)(B/P)u}{1 - \chi(\rho, s)(1 - u)C(\gamma, \theta)}
\]

We can see that this expression admits a vertical asymptote in the space \((u, \frac{w_1}{P})\) when the unemployment rate gets close to \( u = \frac{\chi(\rho, s) - 1}{\chi(\rho, s)} < 1 \) what

\[\text{The firm is assumed to take the decision of other firms as given and to have no influence on aggregate state of the economy.}\]
implies an inferior bound to the definition interval of the unemployment rate. We see that the larger the mark-up on the reservation wage, the higher this inferior bound of the unemployment rate is. This lower bound represents the unemployment trap of this economy, i.e. the minimum unemployment rate that the economy can achieve given the bargaining power of trade unions on the labor market and the competition on the good market. Indeed, if the firm sets the wage by itself \((\rho = 0)\) and if there is perfect competition on the good market \((s \to \infty)\), the mark-up on the reservation will be equal to one and this unemployment trap will be null.

2 Multiple equilibria

In order to deal with the existence of multiple equilibria in this economy, we first analyze some properties of the aggregate equations. Next, we verify the existence of multiple unemployment equilibria and characterize them. To conclude, we discuss the case of non-existence of equilibrium.

2.1 Equations’ properties

For both equilibrium equations we compute the first and second order derivatives to determine their shape, and theirs limits towards the bounds of the unemployment rate’s definition interval.

2.1.1 The aggregate price equation

The first and second order derivatives of the aggregate price equation give us:

\[
\frac{\partial \left( \frac{w}{P} \right)}{\partial u} = (-\alpha \sigma) \frac{(N/F)^{\alpha \sigma}}{s} \frac{C(\gamma, \theta)}{(1-\gamma)^{\alpha(1+\sigma)} \gamma^{\alpha}} (1-u)^{(\alpha \sigma - 1)} \tag{16}
\]

\[
\frac{\partial^2 \left( \frac{w}{P} \right)}{\partial u^2} = (-\alpha \sigma)(1-\alpha \sigma) \frac{(N/F)^{\alpha \sigma}}{s} \frac{C(\gamma, \theta)}{(1-\gamma)^{\alpha(1+\sigma)} \gamma^{\alpha}} (1-u)^{(\alpha \sigma - 2)} \tag{17}
\]

**Proposition 1** The PS curve is always downward sloping in the space \((u, \frac{w}{P})\) whatever the size of the knowledge spillovers in the economy. The PS curve is concave (linear) in the space \((u, \frac{w}{P})\) when \(\alpha \sigma \in ]0; 1[\) \((\alpha \sigma = 1)\), and convex when \(\alpha \sigma > 1\).

**Proof.** We verify that the expression (16) is always negative when \(\alpha \sigma > 0\), and the expression (17) is negative (null, positive) when \(\alpha \sigma \in ]0; 1[\) \((\alpha \sigma = 1, \alpha \sigma > 1)\).

To give an economic interpretation of this proposition we need to understand the consequences of knowledge spillovers, and so of social returns to scale they
involve. When a firm increases its production, it increases its employment level and, as a result, its high-skilled employment level. This involves a rise in the aggregate production and aggregate income which lead to an increase in the aggregate demand. Consequently, the other firms are incited to increase their own output (demand externality coming from monopolistic competition) and in their turn to increase their own high-skilled employment level. Finally, these rises in high-skilled employment level make greater the efficiency of labor used by each firm (knowledge spillovers effect). With a labor more efficient, the firms produce more output with a given employment level what implies a decreasing marginal cost. Thus, the firms set lower prices when the efficiency of labor is increased leading to a lower general price level and higher high-skilled real wage at the aggregate level. That’s why the PS curve is downward sloping in the space \((u, \frac{w_1}{P})\).

The limits computations give\(^6\):

\[
\lim_{u \to u^+} \left( \frac{w_1}{P} \right)_{PS} = \left( \frac{N}{F} \right)^{\alpha \sigma} \left( \frac{s}{s - 1} \left( 1 - \gamma \right)^{\alpha (1 + \sigma)} \gamma \right)^{-1} > 0
\]

\[
\lim_{u \to u^-} \left( \frac{w_1}{P} \right)_{PS} = 0.
\]

These results remain true whatever the size of knowledge spillovers in the economy. The first limit can be interpreted as the maximum real wage that the firms consent to pay in the case where the economy achieves its unemployment trap. For the second one, it’s clear that if all workers are unemployed, there will be no production process and so the firms will pay a null wage.

2.1.2 The aggregate wage equation

The first and second order derivatives of the aggregate wage equation give us:

\[
\frac{\partial (\frac{w_1}{P})_{WS}}{\partial u} = \frac{C(\gamma, \theta) \chi (\rho, s) [1 - \chi (\rho, s)] (B/P)}{\left\{ [1 - \chi (\rho, s) (1 - u)] C(\gamma, \theta) \right\}^2}
\]

\[
\frac{\partial^2 (\frac{w_1}{P})_{WS}}{\partial u^2} = \frac{-2 \chi (\rho, s)}{[1 - \chi (\rho, s) (1 - u)]} \frac{\partial (\frac{w_1}{P})_{WS}}{\partial u}
\]

Proposition 2 When \(\chi (\rho, s) > 1\) and \(u \in [u^-; 1]\), the WS curve is always downward sloping and convex in the space \((u, \frac{w_1}{P})\) whatever the size of the knowledge spillovers in the economy.

Proof. We verify that the expression (18) is always negative and the expression (19) is always positive when \(\chi (\rho, s) > 1\) and \(u \in [u^-; 1]\). ■

Having no influence from knowledge spillovers, the economic interpretation is more usual here. When the unemployment rate increases, the trade union loses

\(^6\) \(u^+\) and \(u^-\) represent the extrem values of the definition interval of \(u \in [u, 1]\).
pressure power on the bargaining. Indeed, given the lack of outside options, its bargaining power is lower what involves a lower high-skilled real wage.

The limits computations give:

\[ \lim_{u \to u^-} \left( \frac{w_1}{\frac{P}{\gamma}} \right)_{WS} = +\infty \quad \text{and} \quad \lim_{u \to u^+} \left( \frac{w_1}{\frac{P}{\gamma}} \right)_{WS} = \frac{\chi (\rho, s) (B/P)}{C(\gamma, \theta)} > 0. \]

The first limit highlights the vertical asymptote that the WS curve exhibits. The second one represents the utility that each high-skilled worker can expect to have in the case of no employment.

### 2.2 Multiple unemployment equilibria

According to the previous properties of the curves, we can state the following proposition about the equilibria existence.

**Proposition 3** When \( \alpha \sigma \in [0; 1] \), there are two distinct unemployment equilibria which belong to \( u \in [u; 1] \).

**Proof.** Let \( \Gamma(u) = \left( \frac{w_1}{P} \right)_{PS} - \left( \frac{w_1}{P} \right)_{WS} \), both equilibria correspond to the roots of this expression. We demonstrate easily that the expression \( \Gamma(u) \) is concave on the interval \( u \in [u; 1] \) when \( \alpha \sigma \in [0; 1] \). It reaches its maximum above the abscissa axis and it cuts this axis twice in the interval \( u \in [u; 1] \) (for more detailed calculations see annex A).

Next, according to the limits of the expressions (13) and (15), we conclude that:

- When \( u \to u^- \), \( \left( \frac{w_1}{P} \right)_{PS} < \left( \frac{w_1}{P} \right)_{WS} \): the PS curve is below the WS curve.
- When \( u \to 1^- \), \( \left( \frac{w_1}{P} \right)_{PS} < \left( \frac{w_1}{P} \right)_{WS} \): the PS curve is below the WS curve.

As a result, we can illustrate the multiplicity of equilibria in this way:
We characterize the two equilibria as follow: one named the "low equilibrium" with a high unemployment rate and a low high-skilled real wage \( \left( u_L, \left( \frac{w_1}{P} \right)_L \right) \) and another named the "high equilibrium" with a low unemployment rate and a high high-skilled real wage \( \left( u_H, \left( \frac{w_1}{P} \right)_H \right) \).

Although a welfare analysis is not executed, we can deduce that the equilibria are pareto ranked with the high equilibrium superior to the low equilibrium. Indeed, in the first one the activity level as well as the real wage is greater than in the second one what implies a higher aggregate demand, production and income, and so a higher welfare of the economy.

2.3 Non-existence of equilibrium

The proposition 3 tells us that it exists two equilibria if and only if \( \alpha \sigma \in [0; 1] \), i.e. if the size of knowledge spillovers is not too large, more specifically if the returns to scale at the aggregate level are not above 2. Indeed, in the reverse case \( \alpha \sigma > 1 \), there is no intersection between the curves and so no equilibrium exists.

As in the Manning’s paper (1990), the absence of equilibrium can be explained by the presence of a hysteresis effect in the unemployment rate. In this case, the unemployment rate follows a random walk and there is no natural rate of unemployment in the economy. Then, the unemployment rate depends mainly on its previous values.
3 Effects of an exogenous increase in the minimum wage

In this section, we analyze the effects of an exogenous increase in the minimum wage on the unemployment equilibrium and the high-skilled real wage thanks to a comparative statics exercise. Then, we find that a policy raising the minimum wage has no effect on unemployment equilibria and reduces the purchasing power of the high-skilled workers. Finally, we compare these findings with those of Card and Krueger (1995) and find some similarities.

3.1 Comparative statics analysis

A policy that implements a rise in the nominal minimum wage is expressed in the model by an increase in the parameter \( \theta \).

**Proposition 4** An increase in the nominal minimum wage has no effect on the unemployment equilibria. Conversely, it involves a decrease in the high-skilled real wage equilibrium that is larger when the economy is at the high equilibrium than when it is at the low equilibrium.

We demonstrate using the Cramer’s rule on the equilibrium system composed of the PS and WS curves that

\[
\frac{du_L}{d\theta} = \frac{du_H}{d\theta} = 0 \quad \text{and} \quad \frac{d\left(\frac{w_H}{P_H}\right)_H}{d\theta} < \frac{d\left(\frac{w_H}{P_H}\right)_L}{d\theta} < 0
\]

(See the annex B for details).

To give an intuition to these results, we may say things in this way. On the one hand, following an increase in the minimum wage, the firms respond to this rise in production cost by a rise in its own prices. Indeed, the partial derivative of (8) according to \( \theta \) shows us that the firm raises its price proportionally about its share of low-skilled employment \( i_7 \). As a consequence, this individual price increase leads to a higher general price level at the general symmetric equilibrium. On the other hand, given the behavior of the trade union, the latter consents to a decrease in the real wage of workers who don’t take advantage of this nominal wage rise in order to keep the employment level unchanged. As a result, the bargaining leads to a lower high-skilled real wage. In this way, at the general equilibrium, the total demand of labor as well as the unemployment rate remains unchanged whereas the high-skilled real wage decreases. Furthermore, at the high equilibrium, the employment level is higher than at the low equilibrium (since the unemployment is lower), so we can deduce that the low-skilled employment level is also higher at the high equilibrium than at the low one. That’s why the high-skilled real wage decrease is greater at the high than at the low equilibrium.

The effect on the general price level can also be deduced by considering the shift of the two curves. The partial derivatives of the PS and WS curves according to the parameter \( \theta \) give us:

\[ \frac{\partial \left(\frac{w_H}{P_H}\right)^*}{\partial \theta} = \left(\frac{\gamma + \gamma}{\gamma - \gamma}\right) \frac{\gamma}{\pi^H(\gamma)} \left(\frac{w_H}{P_H}\right) > 0 \]

7The partial derivative of (8) gives us \( \frac{\partial \left(\frac{w_H}{P_H}\right)^*}{\partial \theta} = \left(\frac{\gamma + \gamma}{\gamma - \gamma}\right) \frac{\gamma}{\pi^H(\gamma)} \left(\frac{w_H}{P_H}\right) > 0 \)
Since \( \frac{w_1}{P} \) at the equilibrium, both curves move in the same extent and in the same direction. These shifts are the result of an increase in the general price level. The expressions (20) and (21) show also that the variation of the general price level is increasing in \( \gamma \), i.e. the proportion of low-skilled labor employment in the total employment. Thus, we can conclude that the increase in the general price level is large enough to cover the new production cost and that an economic policy increasing the minimum wage involves mainly a cost push inflation in the economy.

An other interesting feature of these results is the fact that the implications of this economic policy on unemployment are the same whatever the equilibrium of the economy. Indeed, we have the same comparative statics results at the low and high equilibrium except for the extent of the high-skilled real wage decrease. The equilibrium selection is usually requested in the case where the comparative statics results are reverse to one equilibrium from another to implement an appropriate policy, as in Manning (1990). Consequently, the selection equilibrium analysis is not approached here since an increase in minimum wage will always imply a null effect on unemployment and a cost push inflation.

### 3.2 Comparisons with Card and Krueger’s findings

Card and Krueger (1995) analyze the effects of such a policy by comparing the labor market performances of the 50 US states\(^8\) before and after the 1990 and 1991 increases in the federal minimum wage.

They identify different employment outcomes on the concerned group of workers affected by this rise in the minimum wage in different time periods and regions of the country. There is compelling evidence that the estimated employment effects aren’t significantly different from zero. These rises have no negative employment effect and even sometimes a positive employment effect on the concerned group of workers (Card and Krueger, 1995, pp-389). According to this evidence, the rise in the minimum wage doesn’t imply a necessary rise on unemployment as most models suggest it.

They also analyze the effects of a higher minimum wage on prices in the fast-food restaurants industry which is the leading employer of low-wage workers. Their results show that "the price increases of about the magnitude required to cover the higher cost of labor associated with the rise in the minimum wage" (Card and Krueger, 1995, pp-390).

\(^8\)They divide the states into three groups: two where the wages are high and this increase has little or no effect and one where the wages are low and this increase has important effect.
Another set of their empirical results is about the effect of a higher minimum wage on the distribution of hourly wages. They find that "these increases in the federal minimum wage led to significant increases in wages for workers at the bottom of the wage distribution, and to a reduction in overall wage dispersion" (Card and Krueger, 1995, pp-391).

Our model's results strongly support the two first Card and Krueger's results. As for the effect of a higher minimum wage on the distribution of hourly wages, the similarities with our model are more intuitive. Indeed, we cannot clearly show that the rise in minimum wage increases the wage of workers at the bottom of wage distribution due to the discrete types of labor assumption. Nevertheless, our model allows us to conclude that such a policy leads to a reduction in wage dispersion since the real wage of high-skilled labor decreases and the real wage of low-skilled labor increases or remains unchanged.

The most important discrepancy between these empirical evidences and the common theory concerns the effect of a higher minimum wage on employment. Therefore, Card and Krueger (1995) attempt to give a theoretical explanation of their findings by considering alternative models of labor market from the "textbook" model. They consider models where the wage is either taken by the firm (variants of the "textbook" model) or set by the firm (monopsony model), but never models where the wage of workers paid above the minimum wage results from a bargaining between the firm and a trade union. Machin and Manning (1994), who examine the impacts of a decline in the toughness of minimum wage system on UK's employment, find similar empirical evidences and attempt also to explain theoretically these effects by a monopsony model. Dickens and al. (1999) present a general theoretical model whereby employers have some degree of monopsony and in which minimum wage increases can have positive, neutral or negative effects on employment according to this degree. Thus, our model can be also considered as an additional theoretical explanation of the effects of minimum wage.

Conclusion

In this paper, we have presented an imperfectly competitive model in which multiple unemployment equilibria can occur when the returns to scale of labor are constant at the firm level and increasing at the aggregate level due to the presence of knowledge spillovers in the economy. Then, we have shown that a minimum wage increase raises the general price level as a result of cost push inflation, decreases the real wage of workers who earn more than the minimum wage, and so reduces the real wage dispersion in the economy. Last but not least, we demonstrate that this economic policy has no effect on unemploy-

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9The effect of price increase on real minimum wage depends on the extent of this increase, but in all case it cannot be superior to the extent of nominal minimum wage increase.

10Machin and Manning (1994): "A monopsonistic labor market is one in which an employer possesses some market power in setting wages, so that the supply of labor to the firm is a positive function of the wage paid."
ment equilibria. Since these results come out irrespective of the nature of the equilibrium, we don’t care about the selection problem in this paper.

References


ANNEX

Annex A. Proof of proposition 3

Let \( \Gamma(u) = \left( \frac{w_1}{P} \right)_{PS} - \left( \frac{w_1}{P} \right)_{WS} \) defined on \( u \in [u; 1[ \), with \( \Gamma(u) \in C^2 \). Then, we must show that \( \Gamma(u) \) admits two distinct positive roots in the interval \( u \in [u; 1[ \) when \( \alpha \sigma \in [0; 1[i.e. that \( \Gamma(u) \) cuts twice the abscissa axis in the interval of \( u \in [u; 1[ \) in the space \( (u, \frac{w_1}{P}) \).

\( \Gamma(u) \) is a concave function on the interval \( u \in [u; 1[ \) when \( \alpha \sigma \in [0; 1[ \). Indeed, we have:

\[
\frac{\partial^2 \Gamma(u)}{\partial u^2} = \frac{\partial^2 \left( \frac{w_1}{P} \right)_{PS}}{\partial u^2} - \frac{\partial^2 \left( \frac{w_1}{P} \right)_{WS}}{\partial u^2} < 0
\]

Furthermore, we have:

\[
\lim_{u \to u} \Gamma(u) \to -\infty < \lim_{u \to 1} \Gamma(u) \to -\frac{\chi (\rho, s) (B/P)}{C(\gamma, \theta)}
\]

Now, we just verify that a part of the graph of the function \( \Gamma(u) \) is above the abscissa axis in the interval of \( u \in [u; 1[ \) when \( \alpha \sigma \in [0; 1[ \), so we show that its maximum is positive and belongs to the interval of \( u \in [u; 1[ \).

The polynom \( \Gamma_u(u) = \frac{\partial \left( \frac{w_1}{P} \right)_{PS}}{\partial u} - \frac{\partial \left( \frac{w_1}{P} \right)_{WS}}{\partial u} \) is null when

\[
\left[ \frac{-\alpha \sigma}{(1-u)} - \frac{(1-\chi (\rho, s))}{1-\chi (\rho, s) (1-u)} \right] w_1 P
\]

is null. This polynom has one positive root \( \bar{u} = \frac{(1-\alpha \sigma) [\chi (\rho, s) - 1] - \sqrt{\Delta}}{-2\alpha \sigma \chi (\rho, s)} \), with \( \Delta = (\alpha \sigma - 1)^2 [\chi (\rho, s) - 1]^2 + 4\alpha \sigma \chi (\rho, s) [\chi (\rho, s) - 1] > 0 \), which belongs to the interval \( u \in [u; 1[ \). In addition, we have \( \Gamma(\bar{u}) > 0 \) when \( \alpha \sigma \in [0; 1[ \) and \( \chi (\rho, s) > 1 \). Thus, \( \Gamma(u) \) admits two distinct positive roots in the interval \( u \) when \( \alpha \sigma \in [0; 1[ \) and \( \chi (\rho, s) > 1 \) QED.

Annex B. Proof of proposition 4

Let \( \left( \frac{w_1}{P} \right)_{PS} = \Psi(u, \theta) \) and \( \left( \frac{w_1}{P} \right)_{WS} = \Phi(u, \theta) \), the high equilibrium \( \left( u_H, \left( \frac{w_1}{P} \right)_H \right) \) is solution of this system:
\[ \begin{cases} \Psi(u_H, \theta) - \left( \frac{w_1}{P} \right)_H = 0 \\ \Phi(u_H, \theta) - \left( \frac{w_1}{P} \right)_H = 0 \end{cases} \]

After total differentiation and arrangements, we obtain:

\[ \begin{cases} \frac{\partial \Psi(u_H, \theta)}{\partial u_H} \frac{du_H}{d\theta} + \frac{\partial \Psi(u_H, \theta)}{\partial \theta} - \frac{d \left( \frac{w_1}{P} \right)_H}{d\theta} = 0 \\ \frac{\partial \Phi(u_H, \theta)}{\partial u_H} \frac{du_H}{d\theta} + \frac{\partial \Phi(u_H, \theta)}{\partial \theta} - \frac{d \left( \frac{w_1}{P} \right)_H}{d\theta} = 0 \end{cases} \]

The application of Cramer’s rule and the properties of the PS & WS curves give us:

\[
\frac{du_H}{d\theta} = \frac{\frac{\partial \Psi(u_H, \theta)}{\partial u_H} - \frac{\partial \Phi(u_H, \theta)}{\partial \theta}}{\frac{\partial \Psi(u_H, \theta)}{\partial \theta} - \frac{\partial \Phi(u_H, \theta)}{\partial u_H}} = \frac{\Phi(u_H, \theta) - \Psi(u_H, \theta)}{C(\gamma, \theta)} \]

\[
\frac{d \left( \frac{w_1}{P} \right)_H}{d\theta} = \frac{\frac{\partial \Phi(u_H, \theta)}{\partial u_H} \frac{\partial \Psi(u_H, \theta)}{\partial \theta} - \frac{\partial \Phi(u_H, \theta)}{\partial \theta} \frac{\partial \Psi(u_H, \theta)}{\partial u_H}}{\frac{\partial \Phi(u_H, \theta)}{\partial \theta} - \frac{\partial \Phi(u_H, \theta)}{\partial u_H}} = \frac{\Phi(u_H, \theta) - \gamma}{C(\gamma, \theta)} < 0
\]

At the equilibrium \( u_H \), we have \( \Phi(u_H, \theta) = \Psi(u_H, \theta) \), so \( \frac{du_H}{d\theta} = 0 \) QED. The same argument gives us the same result at the low equilibrium \( u_L \) (\( \frac{w_1}{P} \)).

At the high equilibrium \( u_H \), we have \( \frac{d \left( \frac{w_1}{P} \right)_H}{d\theta} = \frac{\Phi(u_H, \theta) - \gamma}{C(\gamma, \theta)} < 0 \) and at the low equilibrium \( u_L \), we have \( \frac{d \left( \frac{w_1}{P} \right)_L}{d\theta} = \frac{\Phi(u_L, \theta) - \gamma}{C(\gamma, \theta)} < 0 \). Since \( \Phi(u_H, \theta) > \Phi(u_L, \theta) \), \( \frac{d \left( \frac{w_1}{P} \right)_H}{d\theta} < \frac{d \left( \frac{w_1}{P} \right)_L}{d\theta} < 0 \) QED.
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Contact :

Stéphane MUSSARD : mussard@lameta.univ-montp1.fr