« Weak moral motivation leads to the decline of voluntary contributions »

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Abstract

Under the assumption of weakly morally motivated agents, average voluntary contributions decline with repetition of the game, provided that the aggregate moral motivation cannot increase. Our model is compatible with the conditional cooperation hypothesis.

Keywords: Conditional cooperation, voluntary contributions, moral motivation, experiments on public goods games.

JEL:

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1 Introduction

Several experimental studies have documented strong empirical regularities in linear public goods experiments including (1) the fact that people contribute more than predicted by the standard theoretical prediction; and (2) that average contribution declines steadily over time when the game is repeated under a finite horizon.\(^1\) In contrast to the huge amount of literature concerned with the puzzling fact that subjects over-contribute with respect to their Nash contribution, the attempts made to explain the decline of the average contribution observed in most linear public goods experiments are not clear-cut. These attempts boil down to three major explanations: learning, strategic play and other regarding preferences including reciprocity (conditional cooperation).

According to the learning hypothesis over-contributions in early rounds arise because subjects are confused and make errors. As time elapses, and feedback from past rounds becomes available, they realize that they could earn more by over-contributing less, and adjust their current contribution accordingly. However, available evidence about learning suggests that it plays a limited role in

\(^1\)See Ledyard (1995) for a review of this literature published prior to 1995; See also Andreoni, 1995; Croson, 1996; Gaechter & Fehr, 1999; Keser & van Winden, 2000; Fehr & Gaechter, 2000; Masclet et al., 2003; Carpenter, 2007; Sefton, Shupp & Walker, 2007; Herman, Toeni & Gaechter, 2008.
the decay. Neugebauer et al. (2009a) found that repetition without feedback has no effect on average contribution which seems to suggest that there is no learning by introspection. Decay arises only when information about the contribution of group members is provided. In contrast, Houser & Kurzban found a sharper decay when a subject plays against a computer program. Nevertheless, the learning hypothesis seems incompatible with the “restart effect” found by Andreoni (1988).

The second candidate explanation for the decay in average contribution is reputation. The hypothesis of "strategic play" is based on the idea that players take into account future interactions when choosing their current contribution. Therefore, in early periods they have an incentive for establishing a cooperative reputation, by making a large contribution. The justification of the strategic hypothesis is based on the “crazy player” assumption (Kreps et al., 1982), or equivalently on the lack of common knowledge of rationality. If (rational) players believe that there is a crazy player in the group who contributes positively in period 1, it becomes rational for them to play a trigger strategy in early periods and to mix over the strategy space as the repeated game approaches the final period. Andreoni (1988) offered the first test of the reputation hypothesis by comparing the average contributions of partner groups with stranger groups. Since there is no incentive to develop a cooperative reputation among strangers, one should observe higher over-contributions in partner-groups than in stranger-groups, especially in early periods of the repeated game. Surprisingly, Andreoni (1988) found that strangers contribute more than partners, that the difference

2In their computerized treatment human subjects were told that the 3 other members of their group where computerized players, which followed a predetermined contribution path. The latter was chosen to be equal to \( \frac{3}{4} \) of the average contribution observed in the human condition, and the computers’ current choice was announced at the beginning of each round. Overcontribution did not vanish even after 10 rounds. Houser & Kurzban’s conclusion is that more than 50% of the overcontribution is due to confusion.

3After having played 10 rounds of a linear public good game, Andreoni told his subjects that a new sequence of 10 rounds will be played. Surprisingly, he observed that the average contribution of the first restart period does not differ from the average first period contribution in the initial sequence.

4Anderson et al. (2004) developed a more sophisticated theory of learning based on quantal response. Anderson et al. (2004) show that the dynamic process of individual contributions follows the well-known Fokker-Planck equation and converges to the logit equilibrium distribution of contributions. Despite its mathematical elegance, the model has two limitations: first, the model does not explain why contributions are sensitive to the remaining number of periods as observed in partner sessions (due to the neglect of strategic interactions across periods), and second, it assumes that subjects are able to best-respond to the stochastic distribution of other players’ contributions by choosing a stochastic distribution over their strategy space, which requires a high degree of sophistication for each player to form expectations about other players’ choice probabilities. While the model nicely describes the patterns observed in the data, "as if" subjects’ behaviour was of the QRE type, it is not very plausible from a behavioural point of view.

5Strategic play is compatible with the fact that most subjects over-contribute in early periods and switch to their Nash contribution at some later period (see e.g. Isaac et al. (1994), Laury (1997), Keser & vanWinden (2000)). In early periods they signal a desire to cooperate, but as the end of the game approaches their incentive to do so vanishes, and they switch to their Nash contribution. The evidence that the decay is slower in longer games (Isaac et al. 1994) is compatible with the strategic hypothesis.
in average contribution increases over time, and that complete free-riding is significantly more frequent in the partner treatment. These findings seem to destroy the reputation hypothesis as a plausible explanation of the decay in average contributions.\(^6\)

Are other-regarding preferences potential explanations for the decline in contributions? Altruism\(^7\) can account for the fact that people contribute more than their pure self-interested contribution, but cannot explain the decay in contribution. A more plausible explanation is conditional cooperation, i.e. the fact that people choose to cooperate, depending on previously observed decisions of others or on beliefs about their decisions (e.g. Keser and van Winden, 2000; Fischbacher, Gaechter and Fehr, 2001; Croson, 2007; Fischbacher & Gaechter, 2009). Up to recently, reciprocity theories tended to attribute the decay to preference heterogeneity: reciprocal cooperative players are mixed with selfish agents who free-ride on others’ contributions. In a given period, a reciprocal player who either observes that his contribution is above the average, or who expects others to contribute less, will reduce his contribution, and therefore the mean contribution declines. But heterogeneity per se is neither a necessary nor a sufficient condition for the decline.\(^8\) Heterogeneity can either reinforce or attenuate the tendency for the decline, but is not the central driving force of the process. According to Fischbacher & Gaechter (2009) imperfect conditional cooperation is the main driving force behind the decay: "Many people’s desire to contribute less than others, rather than changing beliefs of what others will contribute over time". The intuition behind this is that reciprocators contribute a little less than the observed (or expected) average contribution. Because of such “sticky beliefs” decay can arise even in a population composed exclusively of non-selfish agents. A population of identical non-selfish agents, but slightly selfishly oriented, is enough to provoke the decline. There is strong

\(^6\)Note however that there is no clear evidence that stranger contribute significantly more than partners. Indeed, several studies have also found the opposite, showing that partners contribute significantly more than strangers (see for example Croson, 1996, Fehr & Gaechter, 2000; Masclet et al., 2003). However these studies also indicate that average contribution under stranger matching condition still decline over time as the game is repeated, which cannot be explained by the reputation hypothesis.

\(^7\)One potential explanation relies on the idea that people cooperate because they "take pleasure in others’ pleasure" (see e.g. Dawes and Thaler,1988). Theory of altruism presented by Andreoni and Miller (1996) assumes that an altruistic player’s utility increases not only in his own payoff but also in the other players’ payoffs. Two forms of altruism are generally given in the literature: “pure altruism” (individuals care about others’ payoffs) and “warm-glow” altruism (individuals enjoy contributing per se). However, both pure and impure altruistic motives cannot adequately describe the decline of contribution observed in public goods experiments. Indeed why would altruistic motives vanish over time?

\(^8\)To see why, let us define a perfect reciprocator as a player who matches the average group contribution of the previous period. Consider a case involving only two players: a perfectly reciprocal player and an unconditional player who contributes a fixed amount in each period. As the game is repeated the contribution of the reciprocal player converges to the fixed contribution of the unconditional player, with a slope that depends on the initial contribution of the reciprocator. The example can be easily extended to any mixed population of any finite size composed of perfect reciprocators and unconditional players. Adding noisy players who contribute a random amount does not prevent that the mean contribution either decays or increases.
experimental evidence for such imperfect reciprocity (Fischbacher & Gaechter, 2009, Fischbacher et al., 2001) or selfishly-biased reciprocity (Neugebauer et al., 2009). A conclusion from this short literature review is that imperfect reciprocity is the main driving forces underlying the decay, while learning and strategic behaviour reinforce this tendency.

In this paper, we propose a new model of behaviour, compatible with the imperfect conditional cooperation hypothesis, that accounts for the decline of average contribution. Precisely, it is based on the idea that agents choose their contribution by relying on two dimensions: a "morally ideal contribution" (see Brekke et al., 2003, Nyborg, 2000) and the observed contributions of others. The assumption that people rely on a morally ideal contribution is defended by commitment theories (see e.g. Croson 2007). Based on Kantian reasoning, these theories assume that individuals make "unconditional" commitments to contribute to the public good. Our originality is to assume that for most people such commitments are weak in the sense that they are sensitive to the observation of others' actions. In a contribution context, individuals might therefore be tempted to revise their preferred contribution after observing others' contributions. The extent of such a revision typically varies across individuals: strongly motivated agents will closely stick to their ideal contribution, while weakly motivated agents are prone to revise their morally ideal contribution whenever they observe a gap between their own and others' contributions. Our idea of weakly morally motivated agents captures a large spectrum of contribution behaviours: at one extreme, unconditional contributors, who always stick to their ideal contribution whatsoever, and at the other extreme pure reciprocators who always match the observed group contribution. Free-riding behaviour is a particular case of unconditional behaviour: always contributing zero in a linear public good game whatever the other group members do. Our idea, is that most people are of the "mixed" type, i.e. their actual contribution is the outcome of a deliberative process through which their preferred contribution is balanced against others' observed average contribution. The assumption of weakly morally motivated agents offers a possible justification for imperfect reciprocal behaviour that we call action-based reciprocity: individuals' ideal contribution is sensitive to others' observed contributions. Alternatively, individuals might also decide about their contributions by relying on their expectations about others' contributions (belief-based reciprocity). However, such beliefs are themselves revised according to observed contributions.

The model we propose is primarily designed to explain the decline of average contributions in linear public goods experiments. However, it is also compatible with several other stylized facts observed in experiments on voluntary contributions to a public good. For linear public goods experiments these facts may be summarized as follows: a) subjects contribute half of their endowment in the first period, b) the average contribution tends to decline as the game is repeated, c) there is significant over-contribution in the final period, d) there is high variance of individual contributions, e) most subjects adjust their contribution from one period to the next. Similar observations have been made for interior Nash equilibria and for interior dominant strategy equilibria, except
that stylized fact a) should be understood as “subjects contribute mid way between the equilibrium contribution and the socially optimum contribution” (see Sefton & Steinberg, 1996, Laury & Holt, 1998). Although we are mainly concerned with facts b) and c), our model is also compatible with facts a), c) and d).

Closely linked to our idea, the paper by Kandori (2002) models erosion of norms and morale, to explain various social phenomena including the decay in voluntary contributions. Two recent papers (Ambrus and Pathak, 2007, Klumpp, 2005) propose a dynamic model of voluntary contributions which accounts for the decline. We compare the findings of these papers to our own proposal in the discussion section.

Section 2 introduces our concept of weak moral motivation and shows its implications for the dynamics of average contributions in a simple linear public good model with myopic agents. Section 3 extends the results to non-linear utility and section 4 to non-myopic agents. Our assumptions and results are discussed in section 5 and contrasted with other models. Section 6 concludes.

2 Weak moral motivation and voluntary contributions

Consider $n$ agents, indexed $i = 1, ..., n$, who can contribute voluntarily to a public good. Each of them has an endowment $w_i$, which he can split between his contribution to the public good, $x_i$, and the consumption of private goods, $w_i - x_i$. Using the notation $x_{-i} = \sum_{j \neq i} x_j$, the cardinal representation of agents’ preferences with moral motivation is:

$$U^i(x_i, x_{-i}, \hat{x}_i) = w_i - x_i + \beta_i (x_i + x_{-i}) - v_i (x_i - \hat{x}_i). \quad i = 1, ..., n,$$

where $\beta_i \in [0, 1]$ is the marginal utility from consuming $G = x_i + x_{-i}$, the public good.$^9$

Agent $i$’s moral motivation is embodied in the function $v_i(.)$, where $\hat{x}_i$ stands for his moral obligation. Her loss of utility attached to any deviation from her moral obligation is $v_i(x_i - \hat{x}_i)$. This function is assumed to be convex. In addition two natural assumptions about $v_i(.)$ are as follows:

**assumption 2** $v_i(0) = 0$, $v_i(x_i - \hat{x}_i) > 0$ if $x_i \neq \hat{x}_i$.

$^9$The results of the present paper also hold when preferences are captured by quadratic utility functions:

$$U^i(x_{it}, x_{-it}, \hat{x}_{it}) = \alpha_i (w - x_{it}) - (w_i - x_{it})^2 + \beta_i (x_{it} + x_{-it}) - \frac{v_i}{2} (x_{it} - \hat{x}_{it})^2.$$  

This family of functions, which allows for a dominant strategy equilibrium with strictly positive contributions, has been documented in the experimental literature by relatively few papers (Keser, 1996, Bracht et alii, 2008).
The first assumption is obvious. The second assumption means that, starting from a situation where agent $i$ contributes less (more) than her moral obligation, a marginal increase of $x_i$ reduces (increases) her loss of utility.

We shall conceptualize the weak moral motivation of each agent as a combination of two logics: an autonomous logic and the logic of social influence. The autonomous logic is captured by an ideal, or "ethical", level of contribution noted $x_i^* \geq 0$. For instance, it could correspond to a Pareto optimal level of contributions, i.e. contributing $w_i$ for each $i$. Such autonomous logic can be grounded on a Kantian Categorical Imperative, or on an unconditional commitment to a contribution (Laffont 1975, Harsanyi 1980). The initial contribution is therefore independent of the expected contributions of other players. The second logic captures social influences via the average contribution observed in the immediate past, $\bar{x}_{t-1} \geq 0$. The group contribution is publicly observed after each period. Each player can therefore compare his contribution to the average group contribution. Discovering that her own contribution differs from the group contribution eventually leads her to revise her previous contribution. For example a player who discovers that he has contributed more (less) than the average might decrease (increase) his current contribution. In our model such adjustments towards the previously observed mean contribution are grounded on the revision of agents’ moral motivation.

Overall, the qualified moral obligation, $\hat{x}_{it}$, is defined as a function of the aforementioned variables:

$$\hat{x}_{it} = \begin{cases} x_i^*, & t = 0, \\ M^i(x_i^*, \bar{x}_{t-1}), & t = 1, 2, ... \end{cases}$$

where function $M^i(.,.)$ is discontinuous at $t = 0$, for there is no previous observations at that date that could be used to qualify the autonomous ethical level.

The moral obligation function satisfies the intuitive properties:

**assumption 4** $A_3$: $\frac{\partial M^i}{\partial x_i^*} = M_i^1 \geq 0$, $\frac{\partial M^i}{\partial \bar{x}_{t-1}} = M_i^2 \geq 0$,

**assumption 5** $A_4$: $x_{it}^{\text{min}} = \min(x_i^*, \bar{x}_{t-1}) \leq M^i(x_i^*, \bar{x}_{t-1}) \leq x_{it}^{\text{max}} = \max(x_i^*, \bar{x}_{t-1})$.

Also, it is assumed that the aggregate qualified moral obligation is bounded above by the aggregate autonomous moral obligation:

**assumption 6** $A_5$: $\sum_i M^i(x_i^*, a) \leq \sum_i x_i^* = G^*$, $\forall a \geq 0$. 

6
Because of Assumption (A4), hereafter $M(x_i^*, a)$ is referred to as a *weak moral motivation*. Assumption (A5) plays a key role for the dynamics of contributions in the linear public good’s game, as in (1). According to this assumption aggregate contributions cannot exceed the aggregate initial moral motivation. For instance, if $x_i^* = w_i$ for all $i$, actual contributions are necessarily bounded by aggregate endowment. In a more general sense, assumption (A5) means that players’ moral motivation is not grounded on utopia but on realism and feasibility. However, while (A5) holds in aggregate, it needs not be true at individual levels, i.e., $M^i(x_i^*, a) > x_i^*$ for some $i$ and some $a$ is a possibility. Based on the previous argument it might also be reasonable to assume that $x_i^* \leq w_i$, although we do not require this stronger assumption. Although (A5) is a necessary assumption for the decay in linear public good’s games, we show in section 5, that this assumption is not needed in the case on non-linear utility functions.

**Example 7** An illustration of a weak moral motivation function is the following:

$$\hat{x}_{it} = (1 - \theta_i) x_i^* + \theta_i x_{t-1}$$ \quad $\theta_i \in [0, 1]$ \quad $\theta_i \in [0, 1]$.

The weight $1 - \theta_i$ may be interpreted as the "strength" of agent $i$’s moral motivation. If $\theta_i = 0$ agent $i$ has a strong moral motivation: he never deviates from his ideal contribution, whatever the observed average contribution by other members of his group. On the other hand, an agent for whom $\theta_i$ is close to 1 will strongly revise her initial moral ideal, whenever her current contribution differs from the average group contribution. Assuming the above revision rule, a purely reciprocal player can be defined as a player for whom $\theta_i = 1$, while an unconditional free-rider is defined by $\theta_i = 0$ and $x_i^* = 0$.

If the contribution game is played only once, player $i$ has a dominant strategy to contribute less than his moral motivation. He chooses $x_i$ to solve:

$$\max_{x_{i,t}} w_i - x_i + \beta_i (x_i + x_{-i} - v_i (x_i - x_i^*))$$

The first order condition gives:

$$-1 + \beta_i = v'_i (x_i - x_i^*)$$

At equilibrium the agent equalizes the marginal material cost of a contribution ($\beta_i - 1$) to the marginal moral cost of a deviation from her moral ideal ($v'_i (x_i - x_i^*)$), which implies that her actual contribution is less than her moral ideal:

$$x_i = x_i^* + (v'_i)^{-1} (\beta_i - 1)$$

Since $\beta_i < 1$, $(v'_i)^{-1} (\beta_i - 1) < 0$. 7
While our definition of weak moral motivation is related to a player’s sensitivity to social influence, the above result shows that there is also a "private component" of the weak moral motivation. The latter can be thought as the temptation to deviate from the moral ideal, in order to increase one’s material utility. Indeed the individual chooses her optimum level of contribution by equalizing the marginal material cost of a contribution to the marginal moral cost of deviating from the moral ideal. As we shall see below, this selfish bias towards the material payoff plays also a role in the decay of average contributions.

3 Repeated play with myopic players

Assume now that the contribution game is played a finite number of periods. We assume that in each period players rely on their current updated moral motivation, which is determined by the observed average contribution of the previous period. A key assumption for this section, is that players do not take into account their influence on other players’ future moral motivation when choosing their contribution. We define therefore, for each period of time, a Myopic Nash Equilibrium (MNE) as a profile of contributions such that each agent’s contribution maximizes his own current utility, given the other agents’ contributions:

$$\max_{x_{it}} w_i - x_{it} + \beta_i (x_{it} + x_{-it}) - v_i (x_{it} - \tilde{x}_{it}).$$

From the first order conditions, interior decisions solve:

$$-1 + \beta_i = v_i' (x_{it} - \tilde{x}_{it}), \quad \forall i, \forall t,$$

thus individual equilibrium contributions at period $t$ are:

$$x_{it} = M^i (x^*_i, x_{e_{it}}) + (v_i')^{-1} (\beta_i - 1), \quad \forall i.$$

The revision rule for the moral motivation naturally leads to an interpretation of action-based contributions: current period contributions are partly determined by past observed contributions and partly by individuals’ moral motivation. The revision rule can be easily adapted to capture belief-based reciprocity. If we substitute "others’ average contribution in the previous period" by the "expectation about others’ contributions". The revision rule becomes

$$\tilde{x}_{it} = M^i (x^*_i, x_{e_{it}}),$$

where $x_{e_{it}}$ is agent $i$’s expectation of the average contribution of other players for period $t$. The interpretation is now that agent $i$ determines her current contribution by taking into account her expectation about the (current) contribution of other players and her initial moral motivation. The two formulations do not fundamentally differ if we assume that individuals’ expectations are positively related to their observed contributions of other group members, i.e. if $x_{e_{it}} = f(x_{it-1})$, with $f'(.) > 0$. With this assumption we restrict our interpretation to action-based reciprocity.

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10This concept is not ours. In particular it has been used extensively in the literature on processes (see Drèze and De la Vallée Poussin, 1977, for instance).
Proposition 8: At a MNE, the level of public good is non increasing over time. If Assumption $A_5$ is verified with a strict inequality, then the level of public good is strictly decreasing over time.

Proof. The proof is established recursively. Note first that $G_1 = \sum_i x^*_i + \sum_i (v'_i)^{-1} (\beta_i - 1) \leq \sum_i x^*_i$ because $\sum_i (v'_i)^{-1} (\beta_i - 1) \leq 0$. Then observe that $G_2 = \sum_i M^i (x^*_i, \frac{G_1}{n}) + \sum_i (v'_i)^{-1} (\beta_i - 1) < G_1$, because $\sum_i M^i (x^*_i, \frac{G_1}{n}) \leq \sum_i x^*_i$ by Assumption $A_5$.

Assume the property $G_t < G_{t-1}$ holds for $t = 3, ..., k$, for some $k$. To complete the proof, it must be established that $G_{k+1} < G_k$. This is straightforward, for if the property is true until $t \leq k$, it follows that $G_{k+1} = \sum_i M^i (x^*_i, \frac{G_k}{n}) + \sum_i (v'_i)^{-1} (\beta_i - 1)$ is lower than $G_k = \sum_i M^i (x^*_i, \frac{G_{k-1}}{n}) + \sum_i (v'_i)^{-1} (\beta_i - 1)$, because each $M^i(., .)$ is an increasing function of its second argument and this argument has fallen, $G_k < G_{k-1}$.

The dynamics of aggregate contributions $G_t = \sum_i x_{it}$ are:

$$G_t = \sum_i M^i \left( x^*_i, \frac{G_{t-1}}{n} \right) + \sum_i (v'_i)^{-1} (\beta_i - 1).$$

(2)

The dynamic process in (2) can eventually reach a level of contribution equal to zero, the free-riding equilibrium in standard linear public good’s games. The huge amount of experimental data on voluntary contributions suggests however, that even in the final period, subjects tend to overcontribute significantly. To account for such overcontribution in our framework two additional assumptions are required: the first one stipulates that an increase of the previous level of public good has a less than proportional positive effect on the levels of weak moral motivations:

assumption 9 $A_6$: $M^i_2 \leq 1$.

The second requires the moral motivation to be strong enough to induce a positive level of public good at a MNE even if the previous observable level was zero, i.e. even with an extremely adverse social influence.

assumption 10 $A_7$: $\sum_i M^i (x^*_i, 0) \geq - \sum_i (v'_i)^{-1} (\beta_i - 1)$.

Theorem 11: Under Assumptions $A_5$, $A_6$ and $A_7$, the sequence of public good levels converges to a unique positive interior level $G^\infty \in ]0; G^*]$. 

9
Proof. Under Assumption $A_6$, the right hand side of the dynamics (2) is a contraction. Therefore, according to Banach’s fixed point theorem: i) the dynamics (2) has a unique steady state, ii) the sequence converges towards this steady state. Assumptions $A_5$ and $A_6$ respectively discard the zero and full contributions corner stationary points. ■

**Proposition 12** Under Assumption $A_6$, the higher the autonomous ethical level $x_i$, the higher the long run level of public good $G^\infty$.

**Proof.** The long run level of public good solves

$$G^\infty = \sum_i M_i \left( x_i^*, \frac{G^\infty}{I} \right) + \sum_i (v_i')^{-1} (\beta_i - 1).$$

Using the implicit function theorem:

$$\frac{dG^\infty}{dx_i^*} = \frac{M_i}{1 - \frac{M_i}{I}} > 0$$

under Assumption $A_6$. ■

**Proposition 13** The higher the marginal utility of the public good $\beta_i$, the higher the long run level of public good $G^\infty$.

**Proof.** Using (3) and the implicit function theorem again, one finds:

$$\frac{dG^\infty}{d\beta_i} = \frac{1}{v_i'} > 0,$$

since $v_i(.)$ is a convex function. ■

### 4 Forward-looking players

In this section we relax the assumption of myopic behaviour in order to investigate how forward looking behaviour affects the dynamics of average contribution. Forward looking players take into account the impact of their current contribution on other players’ revised moral motivation in future periods, and are aware that other players try to influence their own moral motivation. Such mutual influence, might eventually lead to an increase in average contributions. We show however, that under reasonable assumptions, the average contribution declines over time.

Let $T$ be the number periods during which agents interacts and $0 < \sigma \leq 1$ their common discount factor. We define players’ intertemporal utility as :

$$\sum_{t=0}^{T} \sigma^t U^i \left( x_{it}, x_{-it}, \hat{x}_{it} \right), \ i = 1, ..., n,$$

where
• \( U^i (x_{it}, x_{it}, \hat{x}_{it}) = w_i - x_{it} + \beta (x_{it} + x_{it}) - v_i (x_{it} - \hat{x}_{it}) \), where \( v_i (x_{it} - \hat{x}_{it}) = \frac{w_i}{2} (x_{it} - \hat{x}_{it})^2 \), \( v_i \) a positive scalar,

• \( \hat{x}_{it} = (1 - \theta_i) x_i^* + \theta_i x_{t-1} = M^i(x_i^*, x_{t-1}), \quad \theta_i \in [0, 1] \).

We will consider a Markov perfect equilibrium (MPE) for this dynamic public good game. Reasoning backward, in the last period agent \( i \) takes as given \( x_{iT} \) and solves:

\[
\max_{x_{iT}} w_i - x_{iT} + \beta_i (x_{iT} + x_{iT}) - v_i (x_{iT} - M^i(x_i^*, x_{T-1}))
\]

He has a dominant strategy, configured by the average contribution inherited from the previous period:

\[
x_{iT} = -\frac{(1 - \beta)}{v_i} + M^i(x_i^*, x_{T-1}) \equiv g_T(x_{T-1}).
\]

It is worth noting that

\[
g_{iT} = M^i_T(\ldots) = \theta_i.
\]

Those equilibrium strategies can be plugged back into the last period utility, giving each agent’s value function for the last period:

\[
V^i_T(x_{T-1}) \equiv U^i \left( g_{iT}(x_{T-1}), \sum_{j \neq i} g_{jT}(x_{T-1}), M^i(x_i^*, x_{T-1}) \right).
\]

Moving backward to the before last period, each agent’s decision solves:

\[
\max_{x_{iT-1}} \left\{ \begin{array}{l}
w_i - x_{iT-1} + \beta_i (x_{iT-1} + x_{iT-1}) - v_i (x_{iT-1} - M^i(x_i^*, x_{T-2})) \\
+ \sigma V^i_T \left( \frac{x_{iT-1} + x_{iT-1}}{n} \right)
\end{array} \right\}
\]

The optimal \( x_{iT-1} \) cancels out the addition of several marginal effects:

i) as when agents are myopic, in the current period:

\[
-1 + \beta_i - v'_i (x_{iT-1} - M^i(x_i^*, x_{T-2})),
\]

ii) but unlike the case of myopic agents, there is also a marginal effect on the next period \( \sigma \frac{\partial}{\partial x_{iT-1}} V^i_T(x_{T-1}) \). This expression can be developed as follows:

\[
\sigma \frac{\partial}{\partial x_{iT-1}} V^i_T(x_{T-1}) = \sigma \left[ -g'_{iT} + \beta_i \left( g'_{iT} + \sum_{j \neq i} g'_{jT} \right) - v'_i (x_{iT-1} - M^i(x_i^*, x_{T-1})) \left( g'_{iT} - M^i_2(x_i^*, x_{T-1}) \right) \right] \frac{\partial x_{iT-1}}{\partial x_{iT-1}}
\]

\[
= \sigma \left[ -g'_{iT} + \beta_i \left( g'_{iT} + \sum_{j \neq i} g'_{jT} \right) \right] \frac{1}{n},
\]

\[
= \frac{\sigma}{n} \left[ (-1 + \beta) \theta_i + \beta \sum_{j \neq i} \theta_j \right].
\]
This second marginal effect explains the difference between the myopic and farsighted behaviors. Note that this difference owes nothing to the next period deviation from the moral motivation. Indeed, a marginal increase of \( x_{iT-1} \) has an impact on the next period moral motivation equal to \( \theta_i dx_{iT-1} \) but this increase is exactly offset by the next period optimal contribution, as noticed from (4), leaving the gap \( x_{iT} - M^i(x^*_i, \bar{x}_{T-1}) \) unchanged. The difference only goes through the effect on the self-centered part \( w_i g^i_{iT}(x^*_i, \bar{x}_{T-1}) \).

Overall, the first order conditions are:

\[
-1 + \beta - v_i' (x_{iT-1} - M^i (x^*_i, \bar{x}_{T-2})) + \frac{\sigma}{n} \left[ (1 + \beta) \theta_i + \beta \sum_{j \neq i} \theta_j \right] = 0 .
\]

Hence, the best (dominant) response for each agent is:

\[
x_{iT-1} = -\frac{(1 - \beta)}{v_i} + \frac{\sigma}{v_i n} \left[ (1 + \beta) \theta_i + \beta \sum_{j \neq i} \theta_j \right] + M^i (x^*_i, \bar{x}_{T-2}) ,
\]

\[
\equiv g_{iT-1} (\bar{x}_{T-2}) .
\]

Observe again that:

\[
g_{iT-1}' = M^i \left( \ldots, \right) = \theta_i .
\]

Each agent’s value function for the before last period is then:

\[
V^i_{T-1} (\bar{x}_{T-2}) \equiv U^i \left( g_{iT-1} (\bar{x}_{T-2}), \sum_{j \neq i} g^j_{iT-1} (\bar{x}_{T-2}), M^i (x^*_i, \bar{x}_{T-2}) \right)
\]

\[
+ \sigma V^i_T \left( \sum_{h=1}^n g^h_{iT-1} (\bar{x}_{T-2}) \right) .
\]

Recursively it is possible to construct the agents’ value functions for each date. There is no conceptual difficulty in this exercise but it is tedious and relegated to the Appendix. The important piece of information is that the individual problems at date \( t \) generically read as:

\[
\max_{x_{iT-t}} \left\{ w_i - x_{iT-t} + \beta_i (x_{iT-t} + x_{iT-t-1}) - v_i \left( x_{iT-t} - M^i (x^*_i, \bar{x}_{T-t-1}) \right) \right\}
\]

\[
+ \sigma V^i_{T-t+1} \left( \frac{x_{iT-t} + x_{iT-t-1}}{n} \right) .
\]

And, using the notation \( \Lambda = \frac{\sigma}{n} \sum_{h=1}^n \theta_h = \sigma \bar{\theta} < 1 \), the dominant strategy \( t \) periods before the last can be written generically:

\[
x_{iT-t} = -\frac{(1 - \beta)}{v_i} + \frac{\sigma}{v_i n} \left[ (1 + \beta) \theta_i + \beta \sum_{j \neq i} \theta_j \right] \sum_{h=0}^{t-1} \Lambda^h + M^i (x^*_i, \bar{x}_{T-t-1} (\bar{\theta}))
\]

\[
\equiv g_{iT-t} (\bar{x}_{T-t-1}) .
\]
In particular, at the first period equilibrium decisions are:

\[ x_{i0} = -\frac{(1 - \beta)}{v_i} + \frac{\sigma}{v_i} \left[ (-1 + \beta) \theta_i + \beta \sum_{j \neq i} \theta_j \right] \sum_{h=0}^{T-1} A^h + x_i^*, \]

\[ = g_{i0}(x_i^*). \]

We are now in a position to investigate whether those contributions could decline over time. Assume that:

**assumption 14 A₈:**

\[-\frac{(1 - \beta)}{v_i} + \frac{\sigma}{v_i} \left[ (-1 + \beta) \theta_i + \beta \sum_{j \neq i} \theta_j \right] \sum_{h=0}^{T-1} A^h \leq 0,\]

\[-(-1 + \beta) \theta_i + \beta \sum_{j \neq i} \theta_j \geq 0.\]

Those two conditions are met for instance when agents value sufficiently the public good (\(\beta\) is large enough) and discount heavily the future (\(\sigma\) small enough). We can then establish:

**Theorem 15** Under assumption A₈, the MPE is characterized by non increasing contributions over time.

**Proof.** Observe first that \(x_{i0} \leq x_i^* \forall i\), by the first inequality in Assumption A₈, hence \(M^t(x_i^*, \pi_0) \leq x_i^* \forall i\). Then, we also have:

\[ x_{i0} - x_{i1} = \frac{\sigma}{v_i} \left[ (-1 + \beta) \theta_i + \beta \sum_{j \neq i} \theta_j \right] A^{T-1} + \left( x_i^* - M^t(x_i^*, \pi_0) \right) \geq 0, \forall i, \]

since, by the second inequality in A₈ the first term in the right hand side of the above expression is positive and, as seen above \(x_i^* - M^t(x_i^*, \pi_0) \geq 0\). Repeating the comparison of successive contributions, one immediately sees that \(x_{it}\) is non increasing over time. \(\blacksquare\)

When the horizon grows large (\(T \to \infty\)), this first equilibrium decisions tend to:

\[ x_{i0} = -\frac{(1 - \beta)}{v_i} + \frac{\sigma}{v_i} \left[ (-1 + \beta) \theta_i + \beta \sum_{j \neq i} \theta_j \right] \frac{1}{1 - A} + x_i^*. \]

Similarly, considering that period \(t\) is the first one, and letting the time horizon go to infinity, it is easy to see that the dominant strategies become stationary feedback rules:

\[ x_{it} = -\frac{(1 - \beta)}{v_i} + \frac{\sigma}{v_i} \left[ (-1 + \beta) \theta_i + \beta \sum_{j \neq i} \theta_j \right] \frac{1}{1 - A} + M^t(x_i^*, \pi_{t-1}), (6) \]

\[ \equiv g_t(\pi_{t-1}), \forall i. \]
Clearly, under Assumption $A_8$, the property of non increasing contributions carries over to the case of an infinite horizon.

A last question is in order: could this logic of behaviors explains the so-called restart effect? The restart effect can be understood in two related yet distinct ways. In the formalism of our model, when the condition of declining contributions is met, one would speak of a restart effect in either of the two following situations:

1. say that the duration of the game is first announced to be of $T/2$ periods, then at date $T/2 - 1$ there is a surprise restart announcement, according to which agents will play a further $T/2$ periods after date $T/2$. there is a restart effect if the the contributions at date $T/2$ with the restart announcement, $x^r_{T/2}$, are larger than the contributions at the same date without the announcement, $x_{T/2}$.

2. imagine now that the announcement is made at date $T/2$; there is a restart effect if the the contributions at date $T/2 + 1$ with the restart announcement, $x^r_{T/2+1}$, are larger than the contributions at date $T/2$ without the announcement, $x_{T/2}$.

Let us investigate each possibility in turn. Without the surprise restart announcement, according to (5) at date $T/2$ agents would contribute:

$$x_i^{T/2} = -\frac{(1 - \beta)}{v_i} + \frac{\sigma}{v_n} \left[ (-1 + \beta) \theta_i + \beta \sum_{j \neq i} \theta_j \right] + M^i \left( x^r_i, x_{T/2-1} \right).$$

With the restart announcement made at date $T/2 - 1$, agents treat the problem as if they were engaged in a new $(T/2 + 1)$-period game, with an initial moral motivation $M^i \left( x^r_i, x_{T/2-1} \right)$, so their contribution in the next period of this new sequence (at date $T/2$) is going to be:

$$x^r_{i,T/2} = -\frac{(1 - \beta)}{v_i} + \frac{\sigma}{v_n} \left[ (-1 + \beta) \theta_i + \beta \sum_{j \neq i} \theta_j \right] \sum_{h=0}^{T/2} \Lambda^h + M^i \left( x^r_i, x_{T/2-1} \right).$$

Comparing the contributions of the two scenarios at period $T/2$, one finds:

$$x^r_{i,T/2} - x_i^{T/2} = \frac{\sigma}{v_n} \left[ (-1 + \beta) \theta_i + \beta \sum_{j \neq i} \theta_j \right] \sum_{h=1}^{T/2} \Lambda^h \geq 0,$$

where the positive sign of the right-hand side is guaranted from the second inequality in Assumption $A_8$. So,

**Proposition 16** A restart effect of the first kind occurs under Assumption $A_8$. 


As for the second kind of restart effect, notice that the contributions in period \( T/2 + 1 \), after the announcement at date \( T/2 \), are going to be:

\[
x'_{i,T+1} = -\frac{(1 - \beta)}{v_i} + \frac{\sigma}{v_i n} \left[ (-1 + \beta) \theta_i + \beta \sum_{j \neq i} \theta_j \right] \sum_{h=0}^{T-1} \Lambda^h + M^i \left( x'_i, \bar{x}_T \right).
\]

Next consider the difference:

\[
x'_{i,T+1} - x_{i,T} = \frac{\sigma}{v_i n} \left[ (-1 + \beta) \theta_i + \beta \sum_{j \neq i} \theta_j \right] \sum_{h=1}^{T-1} \Lambda^h + M^i \left( x'_i, \bar{x}_T \right) - \left[ +M^i \left( x'_i, \bar{x}_{T-1} \right) \right],
\]

\[
= \frac{\sigma}{v_i n} \left[ (-1 + \beta) \theta_i + \beta \sum_{j \neq i} \theta_j \right] \sum_{h=1}^{T-1} \Lambda^h + \theta_i \left[ \bar{x}_T - \bar{x}_{T-1} \right].
\]

Hence

\[
x'_{i,T+1} - x_{i,T} > 0 \iff \frac{\sigma}{\theta_i v_i n} \left[ (-1 + \beta) \theta_i + \beta \sum_{j \neq i} \theta_j \right] \sum_{h=1}^{T-1} \Lambda^h > \bar{x}_{T-1} - \bar{x}_T.
\]

Observe that, under Assumption \( A_8 \), the right hand side of the above inequality is positive (because contributions are decreasing) and bounded. Indeed, the average contributions necessarily fall in the interval \([0, w_i]\), therefore \( 0 \leq \bar{x}_{T-1} - \bar{x}_T \leq w_i \). By contrast, if \( v_i \) approaches zero, the left hand side of the inequality tends to infinity. Thus:

**Proposition 17** Under Assumption \( A_8 \), there exists values of parameters \( v_i \) of the weak moral motivation functions such that a restart effect of the second kind occurs.

## 5 Discussion

In our model the decline of average contributions is driven by two forces. The first one is due to the fact that each agent contributes a little less than his moral ideal because he matches his marginal psychological cost of a deviation from his moral ideal, to his marginal material cost of a contribution. The second is the social influence which leads to a revision of his moral ideal. While the first force always affects negatively his contribution, social influence can have a positive or a negative effect on an agent’s moral motivation. However, positive effects arise
only temporally, because even agents who revise upwards their moral motivation end up reducing their contribution when they hit their budget constraint.

Our model allows for heterogeneity in players’ endowment, preferences, and moral ideal. It therefore encompasses a huge variety of individual behaviours. It predicts most of the observed experimental regularities: over-contributions, heterogeneity of contributions, declining average contributions, final over-contributions and the restart effect. However, it leaves open one important question: do subjects have an initial moral motivation to contribute? Before discussing this issue, we compare briefly our model to other attempts to model the decline. Close to our proposal are the papers by Klumpp (2005) and Ambrus & Pathak (2007).

Ambrus & Pathak (2007) consider a mixed population of players, which consists of selfish and reciprocal types. While the selfish players have homogeneous preferences, there is heterogeneity among reciprocal types, which is captured by reciprocity functions. The dynamics of the model is generated by the behaviour of selfish players who have an incentive to contribute large amounts in early periods because of their influence on future contributions of reciprocal types. The cooperative incentive of selfish players depends on the number of remaining periods in the repeated game. As the end of the game nears, the selfish players switch to their Nash contribution of the one-shot game, i.e. zero contribution. The authors assume a continuous strategy space (players can contribute any real number between 0 and 1). But the key assumption for the decline is common knowledge of preferences of all players, in particular reciprocity functions. Although such an assumption can have some realism in a population of players who know each other well and have experienced frequent interactions over a long period, it does not apply to most experimental data, where subjects are anonymous and interact only for a few periods. Furthermore, the decay in average contribution is obtained by a decline in individual contributions for both types. This requirement seems unnecessarily strong, and does not match individual behaviour in voluntary contribution experiments. The experimental data reveals a high variability of individual contributions from period to period (see e.g. Keser & Van Winden, 2000), which is typically not captured in their model. In contrast, our model allows both for increasing and decreasing individual contributions, with the weaker requirement that aggregate contributions cannot be larger than the initial aggregate moral motivation.

In Klumpp (2005) players are endowed with social preferences. Their utility representation has two additively separable components: material utility and psychological utility. The stage game admits therefore two symmetric Nash equilibria: one where no player contributes to the public good, and one where each player contributes a strictly positive amount. While the dynamic game admits multiple Nash equilibria, the author shows that there is a unique maximal symmetric pure strategies equilibrium path, for which individual contributions decline. As in Ambrus & Pathak (2007) this requirement is too strong for generating a decline in average contributions, and does not correspond to most available data. More important however, is that the temporal profile of the maximal equilibrium path is not compatible with the pattern typically observed in most
experiments on voluntary contributions to a linear public good. In Klumpp’s model, the maximal equilibrium path is one where all players contribute all of their endowment up to some date, after which they start lowering their contribution down to a level that is approximately equal to zero. In contrast, average contributions in linear public goods start at a level that is sandwiched between half the endowment and the Nash contribution (see Laury & Holt, 2008) and then declines slowly to reach a positive level that is significantly larger than the Nash contribution of the constituent game. In Klumpp’s model the average contribution falls very sharply from 100% contribution to nearly 0% contribution over a few periods.

The conclusion of our discussion is that the models proposed by Ambrus & Pathak (2007) and by Klumpp (2005) rely on unnecessarily strong assumptions to generate the decay of the average contribution. Our aim, in this paper was to give a rationale for the decline, based on a reasonable behavioural hypothesis, and accounting for most of the empirical regularities observed in the experimental literature.

6 Conclusion

Our aim in this paper was to provide a general framework that accounts for the decay of the average contribution observed in most experiments on voluntary contributions to a public good. The central idea of our model, is that people rely on two dimensions for taking their contribution decision: a judgement about what is the right contribution, which we called "moral ideal", and the observation of others’ contributions. We defined the combination of these two forces as "weak moral motivation". The decline is generated by two effects, that eventually can be in conflict. The first one is a selfish-bias with respect to the morally ideal contribution. Each player balances her material utility loss from contributing with her psychological utility loss of deviating from her moral ideal. The unambiguous outcome is that each player contributes a little less than her moral ideal. The second - and most important - effect is that an individual’s moral ideal is influenced by the observation of others’ contributions. This effect either increases or decreases the individual’s revised ideal contribution. For example, an individual who observes that his group partners contribute less than himself, will eventually revise downwards her initial moral ideal.

We started by showing that if players behave myopically, i.e. they do not take into account the influence of their contribution on others’ moral motivation, the average contribution is non-increasing as the contribution game is repeated. The stronger result that the average contribution decays is grounded on the crucial assumption that the revised aggregate moral ideal cannot exceed the initial aggregate moral ideal (assumption A5). We acknowledge that at first glance assumption A5 might be considered as a rather strong assumption, but
there are two justifications for this choice. First, the assumption is necessarily satisfied in period 1, since the selfish bias curbs downwards each individual's contribution with respect to her initial ideal contribution. Of course, this fact does not preclude that some players adjust their moral ideal upwards. Assumption A5 can therefore be thought in the following way: downwards adjustments by high-motivated agents always loom larger than upwards adjustment by low-motivated agents. Second, and more generally, it is reasonable to set as an upper limit to the aggregate moral ideal. Budget constraints are on obvious reason. But more importantly, the fact that group interactions occur only over a finite number of periods, sets a natural upper boundary on individual revised moral ideals, whenever these are influenced by social interactions.

The hypothesis of myopic behaviour seems to us as the most appealing with respect to the experimental data about subjects' behavior. However, we cannot preclude the fact that some subjects act as farsighted players and try to manipulate others' moral motivation. Therefore, we provided an extension of our basic model, to account for the more general case of farsighted players. Assuming farsighted behaviour, we showed that the decline arises if agents value sufficiently the public good ($\beta$ is large enough) and discount heavily the future ($\sigma$ small enough). The requirement, that under farsighted behaviour the discount rate must be small with respect to the value of the public good, justifies our preference for the simpler assumption of myopic behaviour for accounting for the decline.

Our behavioural model captures two different motivations for above-Nash contributions and for declining average contributions: reciprocity and moral motivation. While the reciprocity motive has been widely documented in the recent theoretical and empirical economic literature, the moral motivation hypothesis has not retained much attention in the economic literature. There is a large historical trend in the philosophical literature concerned with moral motivation and the strength of will, that is relevant for our assumptions. In our model there is a strong link between these two dimensions. The reason is that reciprocal behaviour is somehow grounded on an internal deliberation process, through which individuals combine their intrinsic motivation to contribute with external pressures in their environment. The hypothesis of weak moral moral can therefore be thought as a means to rationalize reciprocal behaviour. We believe that such hypothesis can be tested with a carefully designed experiment.

Appendix
A Derivation of the Markov Perfect Equilibrium

In the text, equilibrium decisions for periods $T$ and $T-1$ have been given. Moving backward to period $T-2$, each agent’s decision solves:

$$\max_{x_{iT-2}} \left\{ w_i - x_{iT-2} + \beta_i (x_{iT-2} + x_{i-T-2}) - v_i (x_{iT-2} - M^i (x_i^*, \bar{x}_{T-3})) + \sigma V_{iT-1}^i \left( x_{iT-2} + x_{i-T-2} \right) \right\}.$$ 

The marginal effects of changing $x_{iT-2}$ are now as follows:

i) As before there are effects on the current utility:

$$-1 + \beta_i - v'_i (x_{iT-2} - M^i (x_i^*, \bar{x}_{T-3})).$$

ii) there are also marginal effects on the discounted indirect utility of period $T-1$:

$$\left[ \frac{-g'_{iT-1} + \beta_i \left( g'_{iT-1} + \sum_{j \neq i} g'_{jT-1} \right) - v'_i (x_{iT-1} - M^i (x_i^*, \bar{x}_{T-2})) \left( g'_{iT-1} - M_2^i (x_i^*, \bar{x}_{T-2}) \right)}{\sigma} \right] \frac{\partial \bar{x}_{T-2}}{\partial x_{iT-2}},$$

$$= \frac{\sigma}{n} \left[ (-1 + \beta) \theta_i + \beta \sum_{j \neq i} \theta_j \right].$$

iii) and finally, there are marginal effects on discounted indirect utility of period $T$:

$$\left[ \frac{-g'_{iT} + \beta_i \left( g'_{iT} + \sum_{j \neq i} g'_{jT} \right) - v'_i (x_{iT} - M^i (x_i^*, \bar{x}_{T-2})) \left( g'_{iT} - M_2^i (x_i^*, \bar{x}_{T-1}) \right)}{\sigma^2} \right] \frac{\partial \bar{x}_{T-1}}{\partial x_{iT-2}},$$

$$= \frac{\sigma^2}{n^2} \left[ (-1 + \beta) \theta_i + \beta \sum_{j \neq i} \theta_j \right] \sum_{h=1}^{n} \frac{g'_{hT-1}}{n} \frac{\partial \bar{x}_{T-2}}{\partial x_{iT-2}},$$

$$= \frac{\sigma^2}{n^2} \left[ (-1 + \beta) \theta_i + \beta \sum_{j \neq i} \theta_j \right] \sum_{h=1}^{n} \theta_h$$

The first order condition is therefore:

$$-1 + \beta_i - v'_i (x_{iT-2} - M^i (x_i^*, \bar{x}_{T-3})) + \frac{\sigma}{n} \left[ (-1 + \beta) \theta_i + \beta \sum_{j \neq i} \theta_j \right] \left( 1 + \frac{\sigma}{n} \sum_{h=1}^{n} \theta_h \right) = 0.$$ 

And the dominant response can be expressed again as:

$$x_{iT-2} = - \frac{(1 - \beta)}{v_i} + \frac{\sigma}{vn} \left[ (-1 + \beta) \theta_i + \beta \sum_{j \neq i} \theta_j \right] \left( 1 + \frac{\sigma}{n} \sum_{h=1}^{n} \theta_h \right) + M^i (x_i^*, \bar{x}_{T-3}),$$

$$\equiv g_{iT-2} (\bar{x}_{T-3}).$$
The marginal effects of changing \( x_{iT-3} \) are now as follows:

\( i \) the effects on the current utility are:

\[
-1 + \beta_i - v_i' \left( x_{iT-3} - M^i \left( x_i^*, \bar{x}_{T-4} \right) \right),
\]

\( ii \) there are also marginal effects on the discounted indirect utility of period \( T - 2 \):

\[
\begin{align*}
\sigma & \left[ -g_{iT-2} + \beta_i \left( g_{iT-2} + \sum_{j \neq i} g_{jT-2} \right) - v_i' \left( x_{iT-2} - M^i \left( x_i^*, \bar{x}_{T-3} \right) \right) \left( g_{iT-2} - M^i_2 \left( x_i^*, \bar{x}_{T-3} \right) \right) \right] \frac{\partial \bar{X}_{T-3}}{\partial x_{iT-3}}, \\
& = \frac{\sigma}{n} \left[ -1 + \beta \theta_i + \beta \sum_{j \neq i} \theta_j \right],
\end{align*}
\]

\( iii \) and also, there are marginal effects on discounted indirect utility of period \( T - 1 \):

\[
\begin{align*}
\sigma^2 & \left[ -g_{iT-1} + \beta_i \left( g_{iT-1} + \sum_{j \neq i} g_{jT-1} \right) - v_i' \left( x_{iT-1} - M^i \left( x_i^*, \bar{x}_{T-2} \right) \right) \left( g_{iT-1} - M^i_2 \left( x_i^*, \bar{x}_{T-2} \right) \right) \right] \frac{\partial \bar{X}_{T-2}}{\partial x_{iT-3}}, \\
& = \frac{\sigma^2}{n^2} \left[ -1 + \beta \theta_i + \beta \sum_{j \neq i} \theta_j \right] \sum_{h=1}^n \theta_h
\end{align*}
\]

\( iv \) and finally, there are marginal effects on discounted indirect utility of period \( T \):

\[
\begin{align*}
\sigma^3 & \left[ -g_{iT} + \beta_i \left( g_{iT} + \sum_{j \neq i} g_{jT} \right) - v_i' \left( x_{iT} - M^i \left( x_i^*, \bar{x}_{T-1} \right) \right) \left( g_{iT} - M^i_2 \left( x_i^*, \bar{x}_{T} \right) \right) \right] \frac{\partial \bar{X}_{T-1}}{\partial x_{iT-3}}, \\
& = \frac{\sigma^2}{n^3} \left[ -1 + \beta \theta_i + \beta \sum_{j \neq i} \theta_j \right] \left( \sum_{h=1}^n \theta_h \right)^2
\end{align*}
\]

The first order condition is therefore:

\[
-1 + \beta_i - v_i' \left( x_{iT-3} - M^i \left( x_i^*, \bar{x}_{T-4} \right) \right) + \frac{\sigma}{n} \left[ -1 + \beta \theta_i + \beta \sum_{j \neq i} \theta_j \right] 1 + \frac{\sigma}{n} \sum_{h=1}^n \theta_h + \frac{\sigma^2}{n^2} \left( \sum_{h=1}^n \theta_h \right)^2 = 0.
\]
And the dominant response can be expressed again as:

\[
x_{iT-3} = \frac{1 - \beta}{v_i} + \frac{\sigma}{v_i n} \left( -1 + \beta \sum_{j \neq i} \theta_j \right) \left[ 1 + \frac{\sigma}{n} \sum_{h=1}^{n} \theta_h + \frac{\sigma^2}{n^2} \left( \sum_{h=1}^{n} \theta_h \right)^2 \right] + M^i (x^*_i, \overline{x}_{T-4}),
\]

\[
\equiv g_{iT-2} (\overline{x}_{T-3}).
\]

Repeating the logic, and using the notation \( \Lambda = \frac{\sigma}{n} \sum_{h=1}^{n} \theta_h \), the dominant strategy \( t \) periods before the last can be written generically:

\[
x_{iT-t} = \frac{1 - \beta}{v_i} + \frac{\sigma}{v_i n} \left( -1 + \beta \sum_{j \neq i} \theta_j \right) \left[ 1 + \Lambda + \Lambda^2 + \cdots + \Lambda^{t-1} \right] + M^i (x^*_i, \overline{x}_{T-t-1}),
\]

\[
\equiv g_{iT-t} (\overline{x}_{T-t-1}).
\]

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