Mixed strategies in an unprofitable game: an experiment

Charles N. NOUSSAIR, Marc WILLINGER

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Abstract

We report experimental data from a two-player, two-action unprofitable game with an unique mixed strategy equilibrium. Our design allows subjects to explicitly choose probability distributions over actions. Patterns of play differ greatly from the mixed strategy equilibrium and the maxmin strategy profiles, both when measured as subjects’ choices of probability distributions, and as the resulting actions played. The Quantal Response Equilibrium (QRE) concept is a good predictor of the subjects’ average choices.

Keywords: Mixed strategy equilibrium, maxmin, quantal response equilibrium, experimental economics.

JEL Classification : C9, C91, C72.

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1. Introduction

Mixed strategies are a central concept in game theory. In games in which it pays to be unpredictable, choosing a probability distribution over actions is an optimal strategy. Players might mix with the intention of maximizing expected payoff against a player who is doing likewise. The concept of mixed strategy Nash equilibrium (MSE) describes such a situation of mutual best responses. In an MSE, players mix according to a distribution that makes their opponent indifferent between two or more pure strategies. To achieve this, the player relies exclusively on his opponent’s payoffs to calculate the point of indifference. Alternatively, if objective of players is to defend themselves against opponents who intend to minimize their payoff, they would play the Maxmin (MM) strategy profile. In contrast to MSE, the computation of a player’s MM strategy relies exclusively on her own payoffs. Each player chooses the mixture over her set of actions that maximizes her expected payoff, under the assumption that the opponent is intending to minimize it. In some well-known games with unique mixed strategy equilibria, such as matching-pennies or Rock-Paper-Scissors, the MSE and MM strategy profiles coincide. However, in other games the two concepts predict very different outcomes. Such games are of particular interest for investigating how individuals mix over their possible actions, because the two principles can be clearly distinguished from each other.

In this paper we compare how well MSE and MM describes the behavior of experimental subjects playing an unprofitable game. The game is described in section two. In unprofitable games, the rationale for playing MM is particularly plausible. Harsanyi (1966) defines as unprofitable games those in which players can not expect to earn more than their Maxmin payoff in equilibrium. Since a player can always guarantee himself his Maxmin payoff, Harsanyi’s recommendation for players involved in unprofitable games is to choose their Maxmin strategy, rather than riskier Nash strategies. Aumann and Machler (1972) and Aumann (1985) also discuss the implausibility of Nash equilibrium in such games and arrive at the same recommendation.

The only previous experimental study of an unprofitable game that we are aware of is Morgan and Sefton (2002). They studied two unprofitable games that have distinct pure strategy Nash equilibrium and pure strategy Maxmin solutions. They found that neither the Maxmin prediction nor the Nash prediction described the data well. Although there was an incentive to cooperate in that players could a greater total payoff by jointly deviating from their Nash or Maxmin strategies, cooperation was not observed. Rather, Quantal Response Equilibrium (McKelvey and Palfrey, 1995) fit their data better than Nash and Maxmin predictions and the random play hypothesis.
Since we are interested in measuring subjects' mixing probabilities with precision, we use a protocol in which participants play the mixed extension of the game. This allows us to observe explicit mixing on the part of subjects. Rather than choosing their actions directly, subjects are asked to choose probability distributions over their possible actions. After the probabilities are chosen, an exogenous random device chooses the action of each player. While the protocol facilitates explicit mixing because it allows individuals to choose probability distributions, it does not preclude the possibility of conducting the randomization before the choice of probability distribution is made. However, in cases where explicit mixing occurs, the researcher can observe actual randomization, rather than having to infer the existence of randomization from observing a sequence of actions and making the assumption that the actions are drawn from a stationary distribution. In contrast to traditional protocols, it allows for more refined testing of the hypothesis that mixed strategies are used, because it provides additional data: the distributions selected as well as the outcomes of the randomization process that generate the actual actions played.

The Quantal Response Equilibrium (QRE) model has been proposed (McKelvey and Palfrey, 1995) as a general descriptive model of strategy choice in games. One interpretation of model is that each player has an estimate of the expected payoff from each of his actions that includes an unbiased error. Players choose the action that they believe yields the highest expected payoff, given the strategies other players choose. Thus, QRE relies on a different underlying rationale for mixing than do MSE and MM: a strategy is more likely to be chosen the greater its

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1 See Ochs (1995) or Shachat (2002) for other protocols in which probability mixtures are elicited directly.

2 Empirical research in this area has generally focused on the issue of convergence of strategy choices to equilibrium with repeated play. Experimental studies have reached various conclusions about the power of mixed strategy equilibrium to predict behavior when it is the unique equilibrium of a game (see Camerer, 2003, for a survey). Experiments that O’Neill (1989, 1991) and Binmore et al. (2001) report indicate that, for symmetric games, overall choice frequencies are close to the equilibrium predictions. Amaldoss and Jain (2002) report similar findings for asymmetric games. When the mixed strategy equilibrium involves each player choosing each of two actions with equal probability, as in the matching pennies game, behavior is typically consistent with the equiprobable MSE (Mookherjee and Sopher, 1997, Ochs, 1995). Shachat and Swarthout (2003) report evidence that humans readily detect and exploit systematic deviations from equilibrium play in games with a unique Nash equilibrium in mixed strategies. Field studies of data from professional sports (Walker and Wooders, 2001; Chiappori et al., 2002; Palacios-Huerta, 2003; Palacios-Huerta and Volij, 2008) find strong support for the use of equilibrium mixed strategies on the part of professional athletes.

On the other hand, in many other experiments, substantial deviations from the MSE frequencies are observed (Lieberman, 1961; Rapoport and Boebel, 1992; Ochs, 1995; Goeree and Holt, 2000; Shachat, 2002). Furthermore, Brown and Rosenthal’s (1990) reexamination of the data of O’Neill (1989) noted that although overall average choices were close to the equilibrium frequencies, there were a number of serious discrepancies with MSE at the level of the individual decision. Ochs (1995) and Goeree and Holt (2001), among others, illustrate that action choice frequencies depend on own payoffs, and not only on other players’ payoffs, as would be the case in a mixed strategy equilibrium.
expected payoff, yet no strategy is chosen with probability one. Several studies have observed that the direction of deviations from equilibrium in games with a unique MSE is consistent with the predictions of QRE (see for example McKelvey et al., 2000, or Goeree et al., 2003).

There is reason to believe that our protocol would enhance the ability of solutions requiring mixed strategies to describe the data. The protocol facilitates randomization because to generate the appropriate probabilities, subjects do not have to construct random sequences, which are difficult to do in an independent and identically distributed manner. It may also make subjects aware of the potential optimality of mixing. The design also facilitates a focus on the behavioral assumption that underlies the notion of Quantal Response Equilibrium. Because actions with greater expected payoff are played with greater probability, it predicts that behavior in the mixed extension would exhibit the following two properties. Agents would be most likely to play a mixed strategy consisting of placing probability one on the action with the highest expected payoff. The second is that if one of the two pure strategies maximizes expected payoff, a given mixed strategy would be more likely to be observed, the higher the probability it places on the optimal action.

The results of the paper indicate the following. Mixing is widely observed. However, as in the study of unprofitable games of Morgan and Sefton (2002), the observed choices and outcomes are inconsistent with mixed strategy Nash equilibrium, maximin strategies, cooperative behavior and random play. The outcomes are more consistent with Quantal Response Equilibrium than these other four models. The observed mixing does not appear to reflect a desire to be unpredictable, but rather is primarily a consequence of payoff differences between actions. As a result, players earn considerably more than predicted by the Maximin and the Nash solutions. In section 2 we present the theoretical predictions and the experimental procedures. Section 3 presents our results and section 4 concludes with a short discussion.

2. The Experiment

2.1. The Game and Theoretical Models

The game studied is the two-by-two normal form game shown in figure 1. Let $p$ equal the probability that row player chooses the action $U$ and $q$ equal the probability that column player chooses $L$. The game has a unique mixed strategy equilibrium at $p^* = .05$ and $q^* = .05$. We will refer to this strategy profile as the prediction of the MSE. In the MSE, the probability of

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3 Shachat (2002) reports that the availability of an explicit mixing device reduces the autocorrelation in subjects’ decisions from one period to the next.
outcomes $UL$ (Up, Left), $UR$, $DL$, and $DR$ (Down, Right) are $1/400$, $19/400$, $19/400$, and $361/400$. The expected payoffs in the mixed strategy equilibrium are 9.5 for each player.

Figure 1: Normal Form of the Game

<table>
<thead>
<tr>
<th>Outcome of Row Player's Choice</th>
<th>Outcome of Column Player's Choice</th>
<th>LEFT</th>
<th>RIGHT</th>
</tr>
</thead>
<tbody>
<tr>
<td>UP</td>
<td></td>
<td>$190$, $0$</td>
<td>$0$, $190$</td>
</tr>
<tr>
<td>DOWN</td>
<td></td>
<td>$0$, $10$</td>
<td>$10$, $0$</td>
</tr>
</tbody>
</table>

The maxmin solution, at which each player chooses the action where his expected payoff is maximized under the assumption that the other player attempts to minimize his payoff, is at $p^m = .05$ and $q^m = .95$. We will refer to this strategy profile as the prediction of the MM solution. If both players follow their maxmin strategy, which are not mutual best responses for this game, the probabilities of the four outcomes are $19/400$, $1/400$, $361/400$, and $19/400$ for $UL$, $UR$, $DL$ and $DR$ respectively. The expected payoff is 9.5 for each player in the maxmin strategy profile, equal to that in the MSE for all players.

It is clear from figure 1 that there are opportunities to attain total welfare considerably greater than in the MSE or the MM. In the game, the choice of the Row player determines the overall payoff. If $U$ is played, total earnings are 190, but if $D$ is played, they are 10. One simple strategy profile that yields payoffs along the frontier as well as identical expected earnings for the two players is for Row player to choose $U$ and Column player to choose $L$ with probability 0.5. This strategy profile, which we call the Cooperative (CO) solution, yields each player an expected payoff of 95. In a repeated game, the CO outcome can be achieved if Row player plays $U$ in every period, and Column player alternates between $L$ and $R$. The cooperative outcome also corresponds to the Nash Bargaining Solution for the game. The feasible average per-period
payoffs of the game for the two players correspond to a region with vertices at (10,0), (0, 10), (190, 0), and (0, 190). The maxmin payoff vector is (9.5, 9.5). The payoff vector that maximizes the product of the two players’ earnings relative to the maxmin, occurs at (95, 95), the payoff at the cooperative outcome.

Another benchmark is random play, in which each player chooses each of her actions with probability 0.5, or plays each of her strategies of the mixed extension with uniform probability. In both cases, the probability of each of the four possible action profiles is .25, and the expected payoff of each player is equal to 50, considerably greater than either the MSE or MM payoffs.

A Quantal Response Equilibrium to the game is shown in figure 2. The QRE illustrated in the figure assumes the commonly employed logit specification of the relationship between the probability an action is chosen and the error in the estimation of payoffs. The vertical axis indicates the probability that row player chooses $U$ and that column player chooses $L$. The horizontal axis is the level of error, $\lambda$, which corresponds to the probability that $U$ is chosen. The graph for Row player is the solution to $P_U(\lambda) = e^{\frac{\lambda u(U)}{e^{\lambda u(U)} + e^{\lambda u(D)}}}$, where $P_U(\lambda)$ is the probability that action $U$ is chosen under error parameter lambda, and $u(U)$ is the expected utility of action $U$. The other series is analogous, indicating the probability that action $L$ is chosen on the part of Column player. The figure shows that any probability of Row playing action $U$ between .05 and .5988 and any probability of Column playing $L$ between .0064 and .5 is consistent with a QRE. Although a strict interpretation of QRE requires that the strategies of paired Row and Column players correspond to a common $\lambda$, we will require only that each player makes his decision as if the two players have a common $\lambda$, although the actual value of the parameter might be different between the two players.\(^4\) Thus, we will say that any observed values of $p$ and $q$ such that any $p \in [.05, .5988]$ and any $q \in [.0064, .5]$ will be classified as consistent with the QRE solution. Thus the predictions of the QRE solution cover approximately 27.09 percent of the strategy space. If the percentage of outcomes consistent with QRE is significantly greater than 27.09, we will say that the data support the model. The predictions of the four models are summarized in table 1.

\(^4\) McKelvey et al. (2000) take a similar approach in specifying an individual error parameter for each agent to account for the heterogeneous behavior they observe in the games that they study.
Table 1: Predictions of the MSE, MM, CO, RN, and QRE Solutions

<table>
<thead>
<tr>
<th>Solution</th>
<th>Prediction</th>
</tr>
</thead>
<tbody>
<tr>
<td>MSE (Mixed Strategy Equilibrium)</td>
<td>$p^* = .05, q^* = .05$</td>
</tr>
<tr>
<td>MM (Maxmin Profile)</td>
<td>$p^m = .05, q^m = .95$</td>
</tr>
<tr>
<td>CO (Cooperative Model)</td>
<td>$p^c = 1, q^c = .5$</td>
</tr>
<tr>
<td>RN (Random Play)</td>
<td>$p^r = .5, p^r = .5$</td>
</tr>
<tr>
<td>QRE (Quantal Response Equilibrium)</td>
<td>$p^{QRE} \in [.05, .5988], q^{QRE} \in [.0064, .5]$</td>
</tr>
</tbody>
</table>

2.2 Procedures

The experiment was conducted at the Experimental Laboratory of the University Louis Pasteur, located in Strasbourg, France, in November and December 2001. Three sessions, involving 16 subjects each, were organized. Subjects were selected to participate in the experiment by a random draw from the subject pool, which consisted of about 1500 volunteer student subjects from various disciplines in three different universities located in Strasbourg, France. Subjects were randomly assigned either to the role of player A or player B, where player A corresponded to Row and player B to Column. Each player A was randomly matched with the same player B for the entire experiment.
At the beginning of each session, subjects received the instructions, which are given here in the Appendix, and were asked to read them. The experimenter then gave a short verbal summary of the instructions. Afterward, the subjects proceeded through a series of ten questions about the rules of the game that appeared on their computer screens. The questions are given here in the Appendix. If a subject answered a question incorrectly, the computer program stopped, a brief explanation appeared on his screen, and an experimenter assisted him in understanding the correct answer. It was common knowledge that the experiment consisted of exactly 50 periods.

Participants could choose probability distributions over their set of actions. The instructions indicated to subjects that they were endowed with 100 tokens, and that any integer portion of the 100 tokens could be assigned to each of the actions. In other words, in each period participants were required to specify an allocation of 100 tokens between two available actions. The actions were called W and X for Row player and Y and Z for Column player during the experiment. A subject moved a bar on her computer screen to make her choice. Subjects were explicitly reminded in the instructions that they could assign all of the tokens to either one of their actions if they wished to be certain that a particular action would be chosen for them. A random device then chose the actual action played by the subject, where the probability of each action choice was equal to the percentage of the 100 tokens the player had placed on the action. For example, if Row player decided to allocate proportion p of his tokens to action W, that action would be chosen with probability p. This procedure allows explicit choice of the mixing probability, since subjects knew that the allocation in any given period determined their chances of playing each of their two actions. The protocol allows us to compare a predicted probability of each action with the observed probability choice directly. In contrast, in experiments that elicit action choices, implicit probabilities must be inferred from observed outcomes.5

The game was simultaneous so that a player did not know the other player’s decision for the current period until after making his own choice. After subjects had decided on an allocation of their tokens, the outcome was selected at random according to the probability distributions induced by the subjects’ choices. The outcome was announced by displaying on the screen the

5 Ochs (1995) uses a different technique for eliciting a probability distribution over actions. A player participates in a game with two possible actions, A and B. In each period, each player has three options, to play action A, to play action B, or to form a sequence of ten choices of A and B. He is then matched with an opponent in ten identical games, played simultaneously. If he chose action A, he plays A in all ten games. If he chose B, he plays B in all ten games. If he chose to form a list consisting of As and Bs, he plays A in a percentage of the ten games equal to the percentage on his list that were As.

Shachat (2002) uses a system of strategy elicitation similar to ours. In a game with four actions, he allows subjects to place cards of four different colors in a “shoe” in any desired proportion. Each color represented one of the four actions available to the individual. The deck of cards is shuffled and one card is drawn. The color of the card that is drawn determines the action chosen.
option selected for each player and the resulting payoff for both. The payoff matrix, as displayed in the instructions presented to the subjects, is shown in figure 1. The current period earnings of both players and own accumulated earnings until the current point of the experiment were displayed at all times. Subjects could also review the history of play since the beginning of the experiment by hitting a history key. At the end of the experiment, the total amount of Yen, the experimental currency, earned by a subject in the experiment was converted into French Francs at the rate 1 Franc = 20 Yen. Subjects were paid privately one by one, and were invited to write down short comments while waiting their turn to receive payment.

3. Results

The time series of the decisions of each pair of subjects are shown in figures 3-5 below. The figures indicate the number of tokens Row player subjects placed on the upper row, \( U \), and the number of tokens Column player subjects placed on \( L \), by period, for each of the 24 pairs of subjects. In the figures, \( S_j \) denotes subject \( j \) and odd-numbered \( j \) correspond to row players while the even numbers correspond to column players. Players 1 and 2 are paired with each other, as are 3 and 4, etc… The horizontal axes in the figures denote the period number of the session, ranging from 1 to 50. The vertical axes indicate the number of tokens, out of the maximum possible number of 100, that row player placed on \( U \) and column player placed on \( L \).

Several initial impressions can be gained from inspection of the figures and comparison of the data to the predictions of the MSE, MM, and CO solutions. Recall that the MSE predicts an average choice of 5 (5% of all tokens) for both Row and Column player, the MM solution predicts average choices of 5 for Row player and 95 for Column player, and the CO solution predicts a choice of 100 for Row player and an average of 50 for Column player. The average can be attained in several ways. For example, the randomizing device could be used to specify exactly the predicted percentage on each action in any period. Alternatively, a combination of 0 and 100 could be chosen with a frequency that corresponds to the appropriate mixing probability. If randomization occurs before the actual choice of action, identical numbers describe the expected proportion of instances in which each action is chosen. Table 2 illustrates the percentage of instances in which the realized action of each player was \( U \) for Row players and \( L \) for Column players.

The figures illustrate considerable discrepancies between the data and the solution concepts. Overall, the average choice is 45 for Row players and 28.3 for Column players. The average choice of each of the 24 individual Row players as well as each of the 24 Column players
is greater than the equilibrium prediction of 5. The average choice of every Row player is greater than the maxmin prediction of 5, and the average choice of every Column player less than the maxmin prediction of 95. There are two pairs of subjects, players 15 and 16 in session 1, and players 31 and 32 in session 2, that exhibit patterns of behavior that are consistent with the CO for sustained episodes. In particular, the latter pair follows the CO strategy profile perfectly for the first 20 periods. However, overall, it is clear that none of the three solutions provides a satisfactory explanation for the observed data. Furthermore, there appears to be no tendency for decisions to converge in the direction of the predictions of any of the solution concepts with repetition of the game. Thus our first result is that none of these three predictions receive substantial support in our data. The data are differ significantly from random play, as all but three column players play action L in less than 50% of possible instances.

Result 1: Observed choices are highly inconsistent with the Mixed Strategy Equilibrium, the Minimax, the Cooperative Solutions as well as with random play.

Support for Result 1: A t-test rejects the hypothesis that the average choice of Row players (pooling all the choices of all Row players) is equal to 5 ($t = 12.44, p < .001$), indicating inconsistency with the MSE and the MM models. A sign-test rejects the hypothesis that the median (across all Row players) of the average strategy choice of Row players is equal to 5, at $p < .001$, since 24 of 24 players choose an average action greater than 5. The same tests also reject the hypotheses that the mean and median strategies are equal to 100, the prediction of the CO model, at similar significance levels. For Column players, the hypotheses that the average and median strategies chosen are equal to 5, the MSE prediction, and 95, the MM prediction, can all be rejected at the $p < .001$ level. We also reject the null hypothesis of uniform distribution over the strategy space (random play) as is apparent from figure 7, both for row and column player (KS, $p < .01$).

We can also consider whether the proportion of instances in which each outcome, the actual action resulting from a player’s decision, is consistent with the predictions of each solution concept. Using a t-test, we reject the hypothesis that the percentage of instances in which Row’s action is $U$ is equal to 5%, the prediction of the MSE and MM models ($t = 12.33, p < .001$). We also reject the hypothesis that the percentage is equal to the CO prediction of 100 with $t = 16.93$, yielding a similar level of significance. We reject the hypothesis that the percentage of action $L$ outcomes is equal to the MSE prediction ($t = 10.98, p < .001$), as well as equal to the MM prediction ($t = 26.43, p < .001$). A $\chi^2$ test of goodness of fit rejects the hypothesis that the
distribution of the frequency of the four possible outcomes is equal to the MSE prediction ($\chi^2 = 10.26, p < .05$), and the MM prediction ($\chi^2 = 37.44, p < .001$). □
Figure 3: Session 1 Decisions, All Players (horizontal axis: period number; vertical axis: tokens on U or L)
Figure 4: Session 2 Decisions, All Players
Figure 5: Session 3 Decisions, All Players
Table 2: Percentage of choices of U for Row and L for Column

<table>
<thead>
<tr>
<th>Player pair</th>
<th>Session 1</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Row</td>
<td>Column</td>
<td>Row</td>
<td>Column</td>
<td>Row</td>
<td>Column</td>
</tr>
<tr>
<td>1</td>
<td>0.58</td>
<td>0.18</td>
<td>0.38</td>
<td>0.24</td>
<td>0.58</td>
<td>0.38</td>
</tr>
<tr>
<td>2</td>
<td>0.66</td>
<td>0.32</td>
<td>0.34</td>
<td>0.22</td>
<td>0.34</td>
<td>0.52</td>
</tr>
<tr>
<td>3</td>
<td>0.48</td>
<td>0.32</td>
<td>0.30</td>
<td>0.28</td>
<td>0.42</td>
<td>0.24</td>
</tr>
<tr>
<td>4</td>
<td>0.32</td>
<td>0.40</td>
<td>0.24</td>
<td>0.18</td>
<td>0.50</td>
<td>0.60</td>
</tr>
<tr>
<td>5</td>
<td>0.34</td>
<td>0.42</td>
<td>0.36</td>
<td>0.28</td>
<td>0.54</td>
<td>0.50</td>
</tr>
<tr>
<td>6</td>
<td>0.50</td>
<td>0.24</td>
<td>0.36</td>
<td>0.32</td>
<td>0.48</td>
<td>0.26</td>
</tr>
<tr>
<td>7</td>
<td>0.32</td>
<td>0.20</td>
<td>0.24</td>
<td>0.34</td>
<td>0.58</td>
<td>0.08</td>
</tr>
<tr>
<td>8</td>
<td>0.84</td>
<td>0.34</td>
<td>0.78</td>
<td>0.36</td>
<td>0.42</td>
<td>0.32</td>
</tr>
<tr>
<td><strong>Average</strong></td>
<td><strong>0.51</strong></td>
<td><strong>0.30</strong></td>
<td><strong>0.38</strong></td>
<td><strong>0.28</strong></td>
<td><strong>0.48</strong></td>
<td><strong>0.36</strong></td>
</tr>
</tbody>
</table>

The figures also illustrate a tendency for players to increasingly forego the use of the randomizing device over time and to instead make choices of 0 and 100 tokens more often as the game is repeated. Figure 6 illustrates this dynamic over time. Of course, observing a choice of 0 or 100 does not rule out the possibility that randomization occurs mentally before the actual choices are made, and that the pure strategies that we observe reflect the use of mixed strategies. Indeed, the frequent choice of both 0 and 100 on the part of individuals in adjacent periods suggests that some subjects do randomize mentally before making their choice. It is possible that over time, agents become more comfortable making randomizations on their own instead of relying on the device.

Result 2: Use of the explicit randomizing device is widespread but declines over time.

Support for result 2: Figure 6 reveals that 75% of row players and 45% of column players use the explicit randomizing device in period 1. These are therefore lower bounds on the percentage of players that use a mixed strategy, since players may mix without using the device. The figure indicates a tendency for the percentage to decrease over time for the average player. By the last period, the percentages have decreased to 45% and 38% for Row and Column players respectively. □
It is clear that most players change their choices frequently over the course of the game and often employ the explicit mixing device. This suggests that players recognize the need to be unpredictable at least to some extent. To consider the level of predictability of decisions, we estimate the following probit model for each player.

\[
P_t' = \beta_0 + \beta_1 D_{t-1}^{UL} + \beta_2 D_{t-1}^{UR} + \beta_3 D_{t-1}^{DL} + \beta_4 t \tag{1}
\]

\(P_t'\) denotes the number of tokens placed on the player’s first action, \(U\) in the case of the Row player, and \(L\) in the case of the Column player, in period \(t\). The variable \(D_{t-1}^{UL}\) is a dummy variable that equals 1 if the outcome in period \(t-1\) is \(UL\) (which yields a payoff of 190 for Row and 0 for Column), and zero otherwise. \(D_{t-1}^{UR}\) and \(D_{t-1}^{DL}\) are analogous. The estimation is conducted separately for each player in recognition of the obvious heterogeneity in behavior between different individuals. Significant values of \(\beta_1, \beta_2,\) or \(\beta_3\) would indicate a dependence of decisions on the realized outcome of the previous period. Finally, if \(\beta_4\) is significant, there is a general tendency over time for the number of tokens placed on the first action, which is \(U\) for row players and \(L\) for column players, to either increase or decrease, depending on the sign of the coefficient.
The variables included in the estimation equation are those that might be thought to render the player’s decisions predictable from the point of view of the other player. We will interpret the adjusted pseudo-$R^2$ values resulting from estimating the equation for an individual as a measure of the individual’s predictability. The results of the estimation are given in table 3. The table indicates the average and range of adjusted pseudo-$R^2$ values for the 24 players in each of the two roles. It also indicates the number of players for whom the coefficient was significant at $p < .05$ and whether the significance was positive or negative. For example $(1+, 4-)$ indicates that the coefficient on the variable was significantly positive for one of the 24 players in the role indicated, significantly negative for four of the players, and insignificant for the remaining 19 players.

Table 3: Statistics for Estimates of Equation (1)

<table>
<thead>
<tr>
<th></th>
<th>Row Players</th>
<th>Column Players</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average adjusted $R^2$</td>
<td>.148</td>
<td>.157</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>.135</td>
<td>.145</td>
</tr>
<tr>
<td>Range of $R^2$ [min,max]</td>
<td>[006,.600]</td>
<td>[.035,.558]</td>
</tr>
<tr>
<td>$D_{UL}$</td>
<td>1+,4-</td>
<td>1+,8-</td>
</tr>
<tr>
<td>$D_{UR}$</td>
<td>2+</td>
<td>5-</td>
</tr>
<tr>
<td>$D_{DL}$</td>
<td>4+,1-</td>
<td>1+,5-</td>
</tr>
<tr>
<td>Period</td>
<td>2+,4-</td>
<td>2+,9-</td>
</tr>
</tbody>
</table>

Several observations are clear from the data in the table. The first is that the average adjusted $R^2$ for Column players is roughly the same as for Row players, indicating that on average Row and Column players are equally predictable. The second observation is the lack of any variable that is significantly explanatory for more than six of the 24 Row players. The third is the

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6 In addition to the specification presented here, we also considered some others. In particular, we specified $R_{t-1}$ as an independent variable. $R_{t-1}$ equals the number of times during the $t-1$ periods already played that the opponent played his first action, that is, $U$ for Row players and $L$ for column players. We also specified $E_{t-1}$, the earnings difference between the two players from periods 1 to $t-1$, as an independent variable. This takes the form of the sum of the earnings for the opposing player over the $t-1$ periods, divided by the sum of the player’s own earnings for the $t-1$ periods. We also considered $U^{t-1} - U^{t-2}$, the difference in the average payoff that the player has received from periods 1 through $t-1$, between actions 1 and 2. This is calculated for a Row player by averaging his earnings in every period in which action $U$ was chosen from periods 1 to $t-1$, performing the same calculation for the periods in which action $D$ was chosen, and taking the difference between the two averages. For Column players it is the difference between the historical average payoff between actions $L$ and $R$. None of these variables added to the adjusted $R^2$ of most players and they were therefore not included in the results reported here.
lack of a variable that is explanatory for more than eleven Column players. The most pronounced relationship is that nine of the 24 Column players did exhibit a general tendency to play $R$ more frequently over time that was independent of the previous period’s outcome. Some overall patterns are summarized in result 3, which also contains results on the correlation between predictability and earnings.

**Result 3**: Players’ actions are unpredictable. Row and Column players are equally unpredictable on average. Row player predictability is associated with higher average earnings for both players. The level of predictability between an individual and the player with whom he is matched is correlated.

**Support for Result 3**: We cannot reject the hypothesis, using a $t$-test, that the average $R^2$ value over either Row of Column players is equal to zero. We also cannot reject the hypothesis, using a pooled variance $t$-test, that the average $R^2$ is the same between Row and Column players at $p < .05$ ($t = .19$). Taking each player as an observation, there is a correlation of .212 between $R^2$ (predictability) and own earnings for Row players, which is significant at the 5% level. The correlation between predictability and own earnings for Column players was .036, insignificant at the 5% level. For Row players, the correlation between own earnings and the predictability of her partner was -.031. Between Column players’ earnings and Row player predictability, the correlation was .296. The latter is significant at the 5% level. Earnings were significantly higher for Row players who had a predictable partner, but not for Column players. The predictability of Row players and their partners exhibited a positive correlation of .401, significant at $p < .001$, so that the more predictable a player was, the more predictable was his partner. □

The positive relationship between Row player predictability and higher earnings for both players is related to a greater incidence of play of $U$ on the part of Row players, which increases expected total earnings. The fact that Column players choose $L$ more often than in a non-cooperative equilibrium, raises the expected return of playing $U$ and attracts Row players to choose $U$ more frequently. These earnings are not related to the predictability of Column, because Row can be induced to choose $U$ with a predictable strategy of alternating between $L$ and $R$, or an unpredictable mixture that puts sufficient probability on $L$.

Indeed, predictability of Row players appears to be positively correlated with the perceived expected payoff of playing $U$ compared to $D$. In a game with a unique mixed strategy equilibrium such as ours, a player has an incentive to be unpredictable in order to equalize the
expected payoff between the actions of the other player, so that the other player is unable to use a pure strategy best response to a predictable strategy. However inspection of the relationship between a player’s predictability and the difference in the average historical payoff of her two actions suggests a different rationale for the unpredictability we observe. Unpredictability is more likely when the expected payoffs of a player’s own two actions are close to each other. A player is more predictable the larger the difference in his historical average payoff between the two actions. Let \( u^{t-1}_1 - u^{t-1}_2 \) be the difference in the average payoff that the player has received from periods 1 through \( t-1 \) between actions 1 and 2. It is calculated for a Row player by averaging his earnings in every period in which action \( U \) was chosen from periods 1 to \( t-1 \), performing the same calculation for the periods in which action \( D \) was chosen, and taking the difference between the two averages. The variable is calculated in an analogous manner for Column players as the difference between the historical average payoff between \( L \) and \( R \). It seems reasonable to suppose that a player views the historical average payoff of an action as a good predictor of the expected payoff of the action at time \( t \). The pattern suggests the following conjecture.

**Conjecture:** Unpredictability on the part of a player \( i \) is a result of indifference between \( i \)'s own two actions, rather than an attempt to make the other player \( j \) indifferent between his two actions.

**Support for conjecture:** The predictability of player \( i \) is negatively correlated with the difference in the expected payoff of \( i \)'s two actions. The average value over the 49 periods, beginning in period 2, of the variable \( u^{t-1}_U - u^{t-1}_D \) for Row players is 26.24, while the average value of \( u^{t-1}_R - u^{t-1}_L \) for Column players is 57.96. Both \( u^{t-1}_U - u^{t-1}_D \) and \( u^{t-1}_R - u^{t-1}_L \) are greater than zero, when averaged over an entire session, for every pair of players in the study. In an expected payoff sense, every Row player would have been better off playing \( U \) more often and every Column player would have been better off playing \( R \) more often, provided that their partner did not change strategy in response. Among Row players, there is a positive correlation between \( u^{t-1}_U - u^{t-1}_D \) and predictability as captured in the \( R^2 \) term of .482, which is significant at \( p < .001 \). For column players, there is a positive correlation between \( u^{t-1}_R - u^{t-1}_L \) and predictability of .234, which is significant at the \( p < .05 \) level. □

One intuition for why a player would be more unpredictable when the expected payoffs of her two actions are close together is that she perceives the expected payoffs with error. The
smaller the difference in expected payoff, the more likely the subject is to choose a suboptimal action and therefore to appear unpredictable. Such estimation error is one of the underlying behavioral assumptions of Quantal Response Equilibrium. Under QRE, each player’s estimate of the expected payoff of each of his actions is subject to an unbiased error, and the player chooses the action leading to the higher estimated expected payoff. The probability that a suboptimal strategy is chosen is therefore decreasing in the absolute difference between its true expected payoff and that of the optimal strategy. This suggests that QRE may be a good predictor of the patterns we observe in the data. Indeed, as we report in result 4, at the aggregate level, the QRE is quite informative in describing the range of aggregate frequencies of action outcomes that we observe.

**Result 4: Aggregate frequencies of action choices are in a range consistent with the Quantal Response Equilibrium Model.**

**Support for Result 4:** The QRE model allows choices of the Row player, the number of tokens placed on $U$, to be between 5 and 59.88 and those of Column player, the number of tokens assigned to $L$, to be between 0.64 and 50. Although this region covers only 27.09 percent of the space of possible actions, 21 of 24 (87.5%) pairs have average frequencies of outcomes within this range.²

It is instructive to study the three pairs of subjects whose average choices are inconsistent with the QRE. These are pairs 2 and 8 in session 1, and pair 16 in session 2. The source of the failure of QRE in all of these cases is that the Row player played $U$ more frequently than the QRE model allows. These three pairs were also the three pairs who had the highest total payoffs of the 24 pairs in the study. This suggests that group level considerations cause the departure from QRE observed in these groups, and that the assumption of non-cooperative behavior is not fully valid for these groups. This is not surprising given the potential gains from strategy profiles such as the CO solution.

² Therefore, according to Selten’s (1991) measure of predictive success, $S = h - a$, where $h$ measures the hit rate of paired choices falling into the predicted area, and $a$ is the predicted rate, only 87.5% of player pairs have a net positive hit rate.
Figure 7: Distribution of Choices of All Row and Column Players

Figure 7 illustrates the percentage of instances in which each of the 101 possible decisions in the mixed extension, pooled across the players in each role, was played. The vertical column is the percentage of total observations during which the particular number of tokens was placed on $U$ or $L$. It shows that 0 and 100 are the most frequent choices, and that there is a tendency to choose strategies that are divisible by 10 (corresponding to 10% increments), and in particular 50%. There are no choices of action that are not divisible by 5. This is inconsistent with the notion of QRE in the mixed extension, since the probability of the observation of a choice should be increasing in the expected payoff of the choice. Since an action of 0 or 100 always yields the greatest expected payoff, choices of 0 should be more common than 1, which in turn should be more frequent than 2, etc., when 0 is the optimal decision. When a choice of 100 is optimal, 100 should be the most frequent choice, followed by 99, then by 98, etc. In fact, although the strategy with the highest expected payoff is the most common choice, the strategy with the lowest expected payoff, the other (suboptimal) pure strategy, is the second most common. This suggests that in games with a large number of strategies, the QRE model might be supplemented with rules of thumb to narrow down the set of possible choices.

The Nash equilibrium can be calculated for a game close to the mixed extension. Consider a game with 21 actions for Row player and 21 actions for column player. The 21 actions for Row player consist of placing 0, 5, 10, etc… tokens on action $U$. The 21 actions for Column player consist of placing 0, 5, 10, etc… tokens on action $L$. This provides a reasonable approximation to our game since no strategy that was not divisible by 5 was ever chosen in the experiment. The unique Nash equilibrium of the game is the following mixed strategy
equilibrium. Row player places }x\text{ tokens on } U \text{ with probability } P(x) = \frac{1}{2^{\left(\frac{x+5}{5}\right)}}. \text{ In other words, he places } 0 \text{ tokens on } U \text{ with probability } .5, 5 \text{ tokens on } U \text{ with probability } .25, 10 \text{ tokens with probability } .125, \text{ etc… Similarly, Column player places } x \text{ tokens on } U \text{ with probability } P(x) = \frac{1}{2^{\left(\frac{x+5}{5}\right)}}.

The QRE for this 21 action version of the mixed extension also has the property that action 0 is the most frequently chosen action for each player, for any value of } \lambda > 0. \text{ Also for any } \lambda > 0, \text{ the action 5 is the second most frequently chosen, 10 is the third most frequent etc. The data show that 0 is the most common strategy employed for both players, which is consistent with the QRE of the mixed extension. However, as mentioned previously, the data do not show the pattern of monotonic decline in the incidence of play of strategies that involve greater placement of tokens on } D \text{ and } R.

4. Conclusion

The data exhibit the following characteristics. In a manner consistent with previous studies, we find large and qualitative differences between the observed decisions of agents and the mixed strategy equilibrium. Observed decisions are also very different from the maxmin strategy profile, from the strategies that would result from the maximization of total group earnings, and from random decisions. Quantal Response Equilibrium organizes the data better than any of these solutions. These findings are the same as those of Morgan and Sefton (2002), who studied an unprofitable game with Nash equilibrium and maxmin strategy profiles in pure strategies. We show here that their results extend to an unprofitable game with mixed-strategy Nash equilibrium and maxmin solutions.

We also observe a relationship between unpredictability, player roles, and earnings. However, this relationship is quite different than the intuition underlying the concept of mixed strategy equilibrium. Unpredictability appears to indicate indifference between a player’s own available actions, rather the use of a strategy to make the other player indifferent between his actions. Furthermore, unpredictability is not necessarily profitable. For Row players in our game, it was negatively correlated with earnings. Unpredictability appears to have made it more difficult for players to tacitly agree on a cooperative strategy profile.
References


Appendix

This appendix contains a translation from the original French of the instructions given to subjects in the experiment and of the computerized quiz that subjects were required to complete at the beginning of the experiment. The quiz questions are included here for Row Players. Column players were required to complete nearly identical questions, with the terms “player A” and “player B” interchanged and some changes in the earnings figures in the questions to reflect the two different roles.

INSTRUCTIONS

Welcome

The experiment in which you are about to participate is a study of decision making. The instructions are simple. If you follow them carefully and make good decisions, you might earn a considerable amount of money. Your earnings depend on your decisions as well as the decisions of the other subjects in the experiment. All of your decisions will be anonymous and will be transmitted over a computer network. You will indicate your choice at a computer that you will be sitting in front of and your computer will indicate your earnings during the course of the experiment.

Your total earnings for the experiment will be given to you in cash at the end of the experiment.

As soon as all subjects have read through the instructions, one person will proceed to give a summary of the instructions out loud.

Overview of the Experiment

At the beginning of the experiment, you will be matched at random with another subject in this room. For the entire experiment, you will interact only with him or her. The experiment consists of a sequence of periods during each of which you must make a decision. The player you are matched with must also make a decision. During each period, you can earn an amount of money that depends on your choice and the choice of another player. Earnings are expressed in terms of “yen” during the experiment, but your earnings in yen will be converted to francs at the end of the experiment (the procedure for converting yen to francs will be explained at the end of the instructions). There are two types of roles in this experiment, which we will call player A and player B. By a random draw you have been assigned the role of a player ____ and the subject you will interact with has been assigned the role of a player ____.
How the experiment proceeds

The experiment consists of 50 periods. In each period, you must make a choice. To make this choice, you have 100 tokens at the beginning of the period, that you must assign among two options. For player A, the two options are called W and X, and for player B, they are called Y and Z. You must assign all 100 tokens each period. You can choose to assign all of your tokens to one of the two options, or you can assign part of your tokens to one option and the rest to the other option. For example, player A can decide to assign 30 tokens to option W and 70 tokens to option X. Similarly, player B can decide for example to assign 30 tokens to option Y and 70 tokens to option Z.

For player A, the assignment of tokens determines the chance that option W or option X will be realized, according to the following rule: If player A decides to assign N tokens to option W and 100 – N tokens to option X, option W will be selected by the computer with a N in 100 chance and option X will be selected by the computer with a (100 – N) in 100 chance. For example, if player A decides to assign 30 tokens to option W and 70 tokens to option X, there is a 30 in 100 chance that the computer will select option W and a 70 in 100 chance that the computer will select option X. An identical rule applies to the choice of player B. If player B chooses to assign N tokens to option Y and 100 – N tokens to option Z, the chance that the computer will select option Y is N in 100 and the chance that the computer selects option Z is (100 – N) in 100. For example, if player B decides to assign 30 tokens to option Y and 70 tokens to option Z, there is a 30 in 100 chance that the computer will select option Y and a 70 in 100 chance that the computer will select option Z. If a player assigns all of his tokens to one option, it is certain that this option will be selected (because the chance that is selected is 100 in 100).

In each period, the option selected for player A will be matched with the option selected for player B. If the option selected for player A is option W and the option selected for player B is option Y, player A earns 190 yen and player B earns 0 yen. If the option selected for player A is option W and the option selected for player B is option Z, player A earns 0 yen and player B earns 190 yen. If the option selected for player A is option X and the option selected for player B is option Y, player A earns 0 yen and player B earns 10 yen. If the option selected for player A is option X and the option selected for player B is option Z, player A earns 10 yen and player B earns 0 yen.

Table 1 summarizes the possible earnings that players A and B can obtain during a period.
Result of Player B’s Choice

<table>
<thead>
<tr>
<th>Option Y</th>
<th>Option Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>Selected</td>
<td>Selected</td>
</tr>
<tr>
<td>A earns 190 yen</td>
<td>A earns 0 yen</td>
</tr>
<tr>
<td>B earns 0 yen</td>
<td>B earns 190 yen</td>
</tr>
</tbody>
</table>

Result of Player A’s Choice

<table>
<thead>
<tr>
<th>Option X</th>
</tr>
</thead>
<tbody>
<tr>
<td>Selected</td>
</tr>
<tr>
<td>A earns 0 yen</td>
</tr>
<tr>
<td>B earns 10 yen</td>
</tr>
<tr>
<td>A earns 10 yen</td>
</tr>
<tr>
<td>B earns 0 yen</td>
</tr>
</tbody>
</table>

Table 1. Earnings of Player A and Player B

At the time that you make your choice, you do not know the choice made by the other player (that is, his assignment of his or her 100 tokens among his or her two options). Similarly, at the moment the other player makes his or her choice, he or she does not know your choice. After the two players have made their choices for the period, two simultaneous random draws will determine the options selected for the two players: the random draw for player A will determine whether the option selected will be W or X, and the random draw for player B will determine if the option selected will be Y or Z. These two random draws are independent. That is, for player A, the chance that option W or X is the outcome does not depend on the choice of player B, it depends only on the assignment of tokens decided upon by player A. Similarly for player B, the chance that option Y or Z is the outcome does not depend on the choice of player A, it depends only on the assignment of tokens decided upon by player B.

At the end of each period, the computer will inform you of the option that was selected for you, the option that was selected for the other player, your earnings, and the earnings of the other player. All periods will proceed in the same manner.

The total number of yen that you have earned during the 50 periods will be converted to Francs, according to the following conversion rate: 1 franc is equivalent to 20 yen.

Before the experiment begins, you must answer a questionnaire that will be given on your computer, in order to verify your understanding of the instructions.

At the end of the experiment, the experimenter will come to you individually to give you your earnings. While you are waiting for your earnings, you may fill out the comment sheet.

You are asked not to communicate with any other participant during the experiment. If you have a question, raise your hand, and an experimenter will answer your question individually.
Quiz

Questions for Row Players.

True or False?

1. You are matched with the same player in every period.
2. The option selected for a player is determined by a random draw.
3. You cannot be certain whether the option W will be selected by the random draw during a period.
4. If you assign 70 tokens to option W, option X will have a chance of 70 in 100 of being selected.
5. If during a period, the option selected for you is option X and the option selected for player B is option Y, you earn 190 yen and player B earns 190 yen.
6. If during a period, the option selected for you is option W and the option selected for player B is option Y, you earn 0 yen and player B earns 190 yen.
7. At the end of each period, the computer will indicate to you the number of tokens that player B assigned to option Y and Z.
8. At the end of the experiment, if you have obtained accumulated earnings of 1500 yen, your earnings in francs will be equal to 1500*____ francs.
9. The experiment will consist of exactly 50 periods.
10. The decision of the other player in a period will influence the option chosen for you in the period.
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