Comments on "Re-examining the source of Heteroskedasticity: The paradigm of noisy chaotic models"

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Abstract

This paper is a comment on “Re-examining the source of Heteroskedasticity: The paradigm of noisy chaotic models” by Kyriatsou. We summarize their results and discuss some of their conclusion. Simulation was investigated to clarify the functionality of the high dimensional dynamical system and its role in generating process.

Keywords: Noisy chaotic models, Heteroskedasticity, Wavelet transform, self-organized critically.

1- Introduction

In order to examine the source of heteroskedasticity, Blake and Kapetanios (2007) indicates that explicit modeling of the non-linear serial dependence in the time series is likely crucial to correctly understanding its underlying generating mechanism. This non-linear generating mechanism is then recognizably the source of the time variation in both the conditional mean and the conditional variance of the series.

Looking for non-linear serial dependence in conditional mean, several economists have tried to examine the proposition that conditional heteroskedasticity can arise endogenously, as a natural consequence of nonlinear serial dependence in the generating mechanism of the return time series. Such as Ashley (2010), by defining $h$-step-ahead conditional heteroskedasticity, is shown that nonlinear serial dependence in the mean can cause conditional heteroskedasticity only at horizons exceeding one period. This result coincides with those of Kyriatsou (2008), which used both Engel and McLeod-Li tests for detecting the nonlinear dependence in the noisy chaotic models, and concluded that “heteroskedasticity arises endogenously, as a result of interactions between heterogeneous agents in an agent based framework and non linear serial dependency”.

In this paper, we comment on the paper “Re-examining the source of heteroskedasticity: the paradigm of noisy chaotic models”, by Kyriatsou (issue 387, 2008). After a discussion of the implications of their finding, we turn the discussion to the question: is the non linear dependence in noisy chaotic models: heteroskedasticity, spurious or mimic process? Moreover, we interrogate about the functionality of Mackey-Glass equation? These questions have generated some controversy in the last years. For example, Melhem and al. (2010); Rost and Wu (2007); Lani-Wayder and Walther (1996) have written critical reviews on the functionality of the Mackey Glass equation and his role in generating complex dynamics. In order to shed some new light on these important issues we analyse each part of noisy chaotic models using various methods and simulations to check the robustness of the main finding that “…heteroskedasticity in the noisy chaotic models…”

2- Main results

Kyriatsou (2008) use a set of 2 tests to detect nonlinear dependence in the highly dimensional noisy chaos. Nonlinear can occur in the first moment of the process as well as in the conditional variance or even higher moments. The tests suggest the following conclusions:

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2 University of kilaw, kuwait-doha city-block2, kuwait, melhemmahmoud@hotmail.com
3 The heteroskedasticity in the Mackey-Glass model has been signed as a mimic process (Melhem and al. (2010)
• **Endogenous heteroskedasticity**: using a high dimensional noisy chaos mechanism (GMG models), results suggested that nonlinear serial dependence in the mean can cause conditional heteroskedasticity. The importance of this result is that gives rise to new interpretation of markets anomalies and the role of agents’ behavior.

• **Heteroskedasticity**: one form of nonlinear dependence occurs when the conditional variance is time varying. The author considers the Mcleod-Li and Engle tests for dependence in the conditional variance. The result strongly suggests the acceptance of the hypothesis of ARCH behaviors. This is largely in accordance with the widely accepted GARCH effect on the economic and financial time series.

3- **Discussions**

The paper of Kyrtsou (2008) opens the debate on three main important points: the first it’s was largely discussed in Hommes and Manzan (2006), is related to the economic character and modeling. The second is associated with the model dimensions. The third is linked to the outlier. We will discuss these three issues in some more detail.

• **Is the economy characterized high-dimensional noisy chaos?** The author proposed a high dimensional noisy chaos model in thinking that such models have power and better interpretation of the interaction of economics variables and absorb shocks. But this question has been taken a large discussion in the paper proposed by Hommes and Manzan (2006), which simulations concluded that a low dimensional chaos may still explain a significant part of observed fluctuations in economic and financial time series.

• **Dimensions**, the Author set the delay parameter at \( \tau = 1 \). It has been noted that in the case of the Mackey-Glass equation, increasing the value of \( \tau \) also lead to an increase of the dimension of the attractor in chaotic systems (Farmer, 1982). Using a weak value of time delay leads to an appropriate small dimension, which in turn impacts on the measures establishing how jagged the time series is. Subsequently, this phenomenon also affects the values of models optimized results and hence also impacts on conclusions inferred from obtained results.

• **Outliers**, in a pioneer paper of Kyrtsou and Serletis (2006), the estimate of high-dimensional noisy chaos models shows that the extreme observations are inherently related to the dynamical behavior of the model and are not due to large exogenous shocks. Despite that this finding gives a new approach view, some specific literatures in nonlinear and chaos sciences in physical domain announced that there are rarely cases when chaotic events should be considered as outliers. In other hand, in order to identify the kinds of outliers, Xie and al. (2006) showed that outlier patterns are changing with time and its results of both endogenous and exogenous events.

4- **New challenges**

As Kyrtsou (2008) showed that testing for nonlinear dependence in noisy chaotic simulated process suggests acceptance of the hypothesis of ARCH. In this section, we focus, on the functionality of the high dimensional noisy chaotic models. We try to show convincing evidence for presence of unknown structure in generating process of Mackey Glass equation. Moreover, we should show that the interaction between processes may generate a more complex dynamics that can responsible for acceptance of ARCH. To this aim, we should analyse each parts of the noisy chaotic models which can written as

\[
R_t = \alpha \frac{R_{t-\tau}}{1 + R_{t-\tau}^{\delta}} - \delta R_{t-1} + b R_{t-j} \left( 1 - R_{t-j} \right) + \varepsilon_t
\]  

\( (1) \)

\[4\] “Is the Normal Heart Rate Chaotic? Data for study”, National Institute of Biomed. Imag. and Bioeng. 
This model is combine of the Mackey-Glass equation \( \left( \alpha \frac{R_{t-\tau}}{1+R_{t-\tau}^2} - \delta R_{t-1} \right) \), Logistic function \( \left( bR_{t-j}(1 - R_{t-j}) \right) \) and a white noise \( \varepsilon_t \sim N(0,1) \).

### 4.1- Mackey Glass equation

\[
R_t = \alpha \frac{R_{t-\tau}}{1+R_{t-\tau}^2} - \delta R_{t-1}
\]

The Mackey-Glass equation is a simple equation that can generate complex dynamics including chaos. This equation is infinite dimensional systems, which recognized that increasing the value of \( \tau \) increases the dimension of the attractor in chaotic systems. In order to shed some light on this important system, we continue to be advances in understanding the properties of high dimensional system of MG. To this aim, Simulations were performed by generating a number of 5000 observations of low dimensional dynamical systems (Logistic and Lorenz), high dimensional dynamical system (MG) and ARCH models. We then analyse the spectral density based on the Wavelet transform of each simulated system and compare its\(^5\). From figure 1C, it’s possible to read that Mackey Glass model’s computed spectral density shows occupation of all frequency bands. Moreover, surprisingly, the spectral density of this system is established in high frequencies bands with slight amplitude (D1, D2, D3), where ARCH model exist strongly with high amplitude (Figure 1d). While low dimensional systems such the Lorenz and the Logistic function do not exist in such frequency bands (Figure 1b, 1a). Further, we noted that, as the dimensional of the system increases, behaviors transfers from the low to high frequencies and systems become aggressive (Figures 1a, 1b, 1c). Is that means Mackey Glass equation, in generating process, takes slightly into account certain fluctuation that can be classified as conditional heteroskedasticity? This question lead to another question: Is the MG equation can generate another structure else of chaos?

**Figure 1:** Wavelet transform applied on four simulated models (5000 observations)

(a) Logistic function, \( b=3.85, X_0 = 0.1 \).

(b) Lorenz model, \( a=16, b=45,19, c=8/3, X_0 = Y_0 = 0.01, Z_0 = 0.03 \).

(c) Mackey Glass eq., \( \alpha = 2.1, \delta = 0.05, c = 2, \tau = 1 X_0 = 1.2 \).

(d) ARCH model, \( a_0 = 0.2, a_1 = 0.6 \).

In order to illustrate this enigma, we turn back to the definition of MG equation: “The Mackey-Glass equation is a simple equation that can generate complex dynamics including chaos”. Since, we can understand that the MG equation cannot generate a pure chaos

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5 The wavelet transform is commonly used in the time domain. For example, wavelet noise filters are constructed by calculating the wavelet transform for a signal and then applying an algorithm that determines which wavelet coefficients should be modified. Although these coefficients are associated with frequency components, they are modified in the time domain.
structure, and there are exist another structure. This interpreting coincide with the main result of Melhem and al. (2010) where have found that MG equation, in apprehending fluctuations, can generate a dynamical unknown structure that ARCH model can classified as conditional heteroskedasticity and interpreted as nonlinear dependence in the conditional mean. This finding is the extended interpretation of the theoretical results obtained by Röst and Wu (2007) where is shown that “...a proof of chaos in the Mackey-Glass equation has still not been found... and the study of this equation remains a topic of vigorous research”\textsuperscript{6}. Our hence proposed review of the structural composition of the MG equation could be motive to additional research advances that might give way to a more thorough understanding of this equation’s properties. Finally, we can conclude that the presence of both MG and ARCH structures in the same frequency bands may be the underlying explanation for the presence of ARCH dynamics in such systems, as indicated by ARCH test.

4.2- Logistic Function \( R_t = bR_{t-j}(1 - R_{t-j}) \) and simulations

The Logistic function is a low dimensional dynamical system that can express it as autonomy, one-dimensional and nonlinear model. \( b \) is the parameter of control which varies between 1 and 4. In Kyrtou (2008), the author used the logistic function with the parameter of control is 4. In this case, Guegan (2003) showed that when the bifurcation parameter equal to 4, logistic function behave as a white noise. Amplifying the dynamic noise by a similar behaviour has a significant impact on the evolution of nonlinear determinist systems. Further, the dynamic noise may affect the states of the determinist system, and determine its trajectory (Figure 2a, 2b and 2d). Therefore, we interrogate whether interactions between such processes may generate an ambiguous situation whereby one’s behaviour might prevent us from distinguishing that of the other.

In order to shed some light on this issue, simulations were performed by generating a number of 5000 observations for six models presented in Figure 2. In comparing Graphs 2a, 2b, 2c and 2d we note that: as observed, in the absence of noise, it is possible to clearly identify chaotic behaviours in times series. On contrary, when these chaotic behaviours are amplified by combining them with white noise (Graph 2d) resulting process exhibits behaviour over time, similar to that of a random process. Meaning that, as showed above, the white noise affects the low dimensional determinist system. Moreover, the simulated behavior of equation (1) and presented in the Figure (2e) reveals a surprisly finding. We observe that the behavior of equation (1) does not seem of any previous behaviors and it’s appearing that the three structures affect themselves. This behavior is similar to that of an ARCH process but with different amplitude and density. Furthermore, these behaviours exhibit extreme observations which do not exist before and may be results of interactions between processes. This finding coincides with the proposed explanation of Kyrtou and Serletis (2006) which said that outliers are related to the dynamical behaviours. But, did the authors advance their interpretation with respect to such anomalies? This enigma constitutes a serious question on the source of outliers: are the outlier characterized by artificial endogenous or real exogenous shocks? The answer remains an important topic for future works.

Obviously, to get a feel of the influence of the interaction between processes, we verify whether combined processes, in certain cases, are able to disguise as conditional heteroskedasticity. Moreover, we test whether Mackey Glass model exhibit behaviours or mimic behaviours that ARCH may recognized as conditional heteroskedasticity. To clarify

\textsuperscript{6}The existence of chaotic dynamics in delay-differential equations was unknown. Subsequent studies of delay differential equations with monotonic feedback have provided significant insight into the conditions needed for oscillation and properties of oscillations (Walther, 1995; Mallet-Paret-Sell, 1996)
the soundness of this issue, we proceed as Kyrtou (2008) using the Engle test to detect the nonlinear dependence on each of simulated series. Bootstrapping simulations are carried out in table 1, based on 1000 replication and $T = 5000$ of each times series. It’s not surprisingly that the Engle test rejects the hypothesis of ARCH behaviors in both Logistic function and noisy logistic function. But, the Engle test applied on the Mackey Glass simulated time series suggest that acceptance of the hypothesis of ARCH behavior at 5% significance level. Meaning that’s Mackey Glass equation has a conditional heteroskedasticity behavior. To support our analysis, we reapply ARCH test on amplified series of equation (1) to detect presence of heteroskedasticity in the structure. We note that ARCH test displays statistically significant parameters at 5% level. Here, one could interrogate whether the conditional heteroskedasticity arises endogenously, why the Engle test reject the hypothesis of ARCH for low dimensional noisy chaos and accept it for high dimensional noisy chaos?

![Figure 2: Trajectory simulations of six models (5000 observations).](image)

(a) Logistic Function trajectory $b=4, X_0 = 0.1$.  
(b) White Noise trajectory

(c) MG trajectory $\alpha = 0.3, \delta = 0.05, c = 10, \tau = 1$ and $X_0 = 0.01$  
(d) Logistic function plus white noise trajectory

(e) Mackey Glass plus logistic function plus white noise trajectory  
(f) ARCH trajectory $\alpha_0 = 0.2, \alpha_1 = 0.6$

<table>
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<tr>
<th>Table 1: Significance level for the Engle test</th>
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<tbody>
<tr>
<td>Bootstrap</td>
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<tr>
<td>Logistic Function</td>
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<tr>
<td>Mackey Glass</td>
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<tr>
<td>Logistic + White Noise</td>
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* Acceptance of the hypothesis of ARCH at 5% significance level. ** 10% significance level.

5- Conclusion

Several papers showed that as conditional heteroskedasticity is often ascribed to stochastic volatility models, the evidence also showed that is arises endogenously is strong. But, it is not clear which noisy high nonlinear deterministic mechanism offers the best interpretation for
this detected behavior, and this remains an important topic for future researches. Our simulations show that the high dimensional dynamical system may exhibit nonlinear serial dependence behaviors in the generating mechanism of the return time series but, structurally, it is not chaotic. One can explain that the Mackey Glass equation can generate, beside a chaos, another unknown structure that ARCH may recognized as conditional heteroskedasticity. Another can interprets as mimic behaviours. Hence, our proposed review of the structural composition of the MG equation could be motive to additional research advances that might give way to a more thorough understanding of this equation’s properties. Therefore, we must be careful in interpreting its results.

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