« Less Risk, More Effort:
Demand Risk Allocation in Incomplete Contracts »

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DR n°2012-20
LESS RISK, MORE EFFORT: DEMAND RISK ALLOCATION IN INCOMPLETE CONTRACTS∗

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June 19, 2012

Abstract

This article investigates the allocation of demand risk within an incomplete contract framework. We consider an incomplete contractual relationship between a public authority and a private provider (i.e. a public-private partnership), in which the latter invests in non-verifiable cost-reducing efforts and the former invests in non-verifiable adaptation efforts to respond to changing consumer demand over time. We show that the party that bears the demand risk has fewer hold-up opportunities and that this leads the other contracting party to make more effort. Thus, in our model, bearing less risk can lead to more effort, which we describe as a new example of ‘counter-incentives’. We further show that when the benefits of adaptation are important, it is socially preferable to design a contract in which the demand risk remains with the private provider, whereas when the benefits of cost-reducing efforts are important, it is socially preferable to place the demand risk on the public authority. We then apply these results to explain two well-known case studies.

Keywords: Public-Private Partnership, Incomplete Contract Theory, Contractual Design, Demand Risk, Counter-Incentives

JEL codes: D23; H11; L33

∗The authors are grateful to Jean-Etienne de Bettignies, Eschen Chong, Eduardo Engel, Antonio Estache, Jose Luis Guasch, Mario Jametti and David Martimort for their helpful comments and suggestions. We also benefited from comments made by participants at the "Contracts, Procurement, and Public-Private Arrangements" conference in Paris, May 2011, at the "Public Private Partnership" conference in Barcelona 2011, at the ISNIE conference 2011, at the IESE conference, April 2012, and at IDEP, Lugano, April 2012.

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1 Introduction

Public-private partnerships (PPPs) have increased in popularity over the past couple of decades across a broad range of public services (e.g. roads, bridges, schools, hospitals, prisons). PPPs, which bundle construction and operation into a single contract, encompass a variety of administrative arrangements (Grout and Stevens, 2003). Nevertheless, we can distinguish between two main types of contract used to delegate public services to private operators: (i) contracts where private providers bear no demand risk (e.g. they receive service payments from the procuring authority according to performance criteria) and (ii) contracts where private providers bear all the demand risk (e.g. payment depends on the actual service demand). Although the traditional contract model of PPPs used around the world has been the latter (i.e. demand risk remains with the private provider), a number of countries have recently promulgated guidelines that encourage the former (e.g. the June 2004 act in France instituting the new contrats de partenariat).

There is some consensus that contracts in which private providers bear no demand risk are useful in two scenarios: (i) when it is not possible to make users pay (on technological, social or institutional grounds) and (ii) when the services are unprofitable. However, in practice, certain contracts specify that the service provider be remunerated according to service demand even if users do not pay (these are often termed ‘shadow toll contracts’). By contrast, in other cases, the remuneration of the service provider depends on the continuity of supply of the service while users pay a toll (to the public authority). Thus, it would appear that the choice between a contract in which the private provider bears demand risk and one in which he does not depends neither on the ability to make users pay nor on the profitability of the service in question. This notion implies the futility of making the remuneration of the private provider dependent on service demand when there is no real demand, such as for prisons, or when demand is inelastic, such as in the defence sector or for public lighting. Therefore, the types of public services explored in this paper are transport infrastructure (e.g. bridges, roads, urban transport, car parks), garbage collection, leisure centres (e.g. stadiums), schools and hospitals.

This general background raises the issue of the allocation of demand risk in PPPs. The present paper addresses this issue by comparing the efficiency of the foregoing types of contract from an incomplete contract perspective (Grossman and Hart, 1986; Hart and Moore, 1990; Hart, 1995; Hart, Shleifer and Vishny, 1997; Hart and Moore, 1999; Hart, 2003). An incomplete contract, which is a contract that does not account for all the relevant variables, as opposed to a complete contract which does, may apply because the variables that pertain to its fulfilment are not verifiable (Hart, 1995). Thus, according to Hart and Moore (1999), an incomplete contract is one in which “the parties would like to add contingent clauses, but are prevented from doing so by the fact that the state of nature cannot be verified (or because states are too expensive to describe ex ante).”

In light of the foregoing, this paper considers a contractual relationship in which a public authority contracts with a private provider.\textsuperscript{1} The private provider invests in non-contractible cost-

\textsuperscript{1}We herein refer to the public authority as ‘she’ and to the private provider as ‘he’.
reducing efforts that are specific to this PPP while the public authority makes a non-verifiable investment, which corresponds to the effort of adapting the public service provision over time to the changing consumer demand (also called the ‘demand-enhancing effort’). This assumption contrasts with the literature on PPPs, which has mainly analyzed PPP contracts in which the private provider is responsible for all investment. However, in line with Ellman (2006) and Athias (2009), we consider that public authorities play an active role in adapting the private provision of public services over time.

In both types of contract, the private provider has the control rights over the service provision, and renegotiation should therefore take place to allow the adaptation required by the public authority to be implemented. Either party can hold-up the other and the incentives of the contracting parties are driven by the (anticipated) outcome of the (efficient) renegotiation to share the non-contractible surplus generated by the non-verifiable efforts. We assume that the private provider faces the risk of bankruptcy when he bears demand risk, which is not the case for the public authority. We show that the trade-off turns out to be a choice between the public authority holding-up the private provider, or vice versa. In summary, we show that the party that bears the demand risk has fewer hold-up opportunities and that this leads the other party - that does not bear the demand risk - to make more effort.

The model leads to two main results. Firstly, the investment of the public authority in adaptation is lower when she bears the demand risk. Secondly, the cost-reducing efforts of the private manager are less when he bears the demand risk. A trade-off therefore occurs between adaptation and cost-reducing effort in the allocation of demand risk. Further, we find that when the benefits of adaptation are important, it is socially preferable to design a contract in which the demand risk remains with the private provider, whereas when the benefits of cost-reducing efforts are important, it is socially preferable to place the demand risk on the public authority. We apply these results to two well known case studies: the case of a contract in which the demand risk remains with the public authority (the British school catering case), and the case of a contract in which the demand risk remains with the private provider (the French highway concession case).

The present paper makes two important contributions to the literature. Firstly, the theoretical results are counter-intuitive and at variance with those typically derived when investigating the moral hazard problem using agency theory (Iossa and Martimort, 2008 and Iossa and Martimort, forthcoming). Indeed, according to agency theory, the agent increases his efforts when he bears more risk. By contrast, the present paper highlights a new case of ‘counter-incentives’, a phenomenon relatively scarce in the literature (e.g. Lazear, 1989; Benabou and Tirole, 2006; Winter, 2009). Secondly, our model is the first to highlight the fact that beyond the theory of incentives, a framework of incomplete contracts is useful for analyzing the allocation of risk in contracts.

The remainder of this paper is organized as follows. Section 2 describes the relevant literature and states the contributions of this paper. Section 3 presents the model, and our theoretical propositions are developed in Section 4. In Section 5, the model is applied to explain two case studies, and Section 6 concludes.
2 Literature Review

2.1 Risk allocation in contract theory

According to the well developed agency theory with moral hazard, incentives and risk are bundled together (see Tirole, 1988; Laffont and Martimort, 2001; and for the case of a PPP see Iossa and Martimort, 2008 and forthcoming). In other words, the greater the incentive provided by the principal to the agent, the more mechanically risky the payoff of the agent. In fact, because the effort of the agent is unobservable, the contract is based on the stochastic level of production, which is supposed to be observable. A contract provides strong incentives to the agent only if his remuneration depends significantly on production. Because production is stochastic, stronger incentives mean a great degree of risk (to the agent). As a consequence, the effort of the agent increases at the same time as his payoff becomes more risky; this result holds whether or not the agent is risk neutral.

By contrast, we find the opposite result under an incomplete contract model. In this framework, the ability to renegotiate enables the parties to adapt the contract to states of nature as they unfold. Thus, the incompleteness of contracts makes direct incentive contracts unfeasible; rather the incentives are derived from the renegotiation of the contract. In other words, in incomplete contracts incentives are driven by the (anticipated) outcome of the ex post (efficient) renegotiation for sharing the surplus generated by non-verifiable efforts.

Given that residual rights (which follow ownership) significantly influence the outcome of this renegotiation, the works of previous authors on incomplete contracts have focused on the ownership decision (Hart, Shleifer and Vishny, 1997; Hart, 2003; Bennett and Iossa, 2006). However, the present paper uses the same -complete contract- framework to analyze risk allocation in contracts. In particular, we show that to bear more risk implies fewer opportunities for hold-up, which leads the party that bears less risk to make more effort.

2.2 Efforts in PPPs: Empirical Evidences

Although many previous studies have analyzed PPP contracts in which the private party is responsible for all investments (e.g., Hart, 2003), Ellman (2006) and Athias (2009) considered the active role of public authorities. These authors argued that public authorities play an important role in the adapting the private provision of public services over time for the following three reasons. Firstly, any PPP is a joint relationship between a public authority and a private provider (and as such the public cannot vote to oust the latter). Secondly, private providers are not accountable to the market because any price applied to consumers is regulated, rather than a market price. Finally, public authorities, as the elected delegates of consumers, are duty bound to place pressure on the private provider to adapt the public service to meet changes in consumer demand. In other words, studies of the efficiency of PPPs must consider public authorities to be proactive rather than passive bystanders.

The experience of the British government with school dinners offers a good example of the role
played by public authorities in improving the efficiency of PPPs (case study 1). As highlighted by Ellman (2006), in the aftermath of a series of television reports on school diners by celebrity chef Jamie Oliver in early 2005, the government rushed to satisfy mounting public discontent over low quality by committing to make improvements in all schools, including those whose catering services were provided through PPP contracts. In this case, we may observe that the private provider invested in cost-reducing efforts, while the procuring authority pressured the private provider to adapt his service according to the demand for healthy food from the public.

Another good example of the efforts of the public authority in PPPs is provided by the case of the “Shipwrecked Men of the Road of Saint-Arnoult-In-Yvelines”. In 2003, the French meteorological office underestimated the extent of the snowfall across central and northern areas of France. Consequently, the private provider charged to manage the highways in these areas failed to take all the necessary measures to preserve the viability of the junction of two roads. The private provider failed to establish diversions or give information, which caused traffic chaos within a radius of 60 km of the problem. After this event, there was public discontentment about the lack of effective provision in the case of heavy snowfall. Therefore, the French government pressured the private provider to adapt his service provision by investing in lighter gritting vehicles and automatic gritting systems.

For simplicity, we voluntarily omit the potential role of the private provider in discovering adaptation in our model. This is not to deny its importance (Hart, Shleifer and Vishny, 1997; Besley and Ghatak, 2001; Hart, 2003; and Bennet and Iossa, 2006); however, if we assume that the benefits of the adaptation investments are additively separable, the adaptation investment equilibrium of the public authority is not affected by the adaptation investment of the private provider.

3 The Model

As introduced above, there are two main types of contract for delegating public services to private operators: (i) contracts where private providers bear no demand risk (e.g. they receive service payments from the procuring authority according to performance criteria) and (ii) contracts where private providers bear all the demand risk (e.g. payment depends on the actual demand for the service). Both types are long-term global contracts for the design, building, financing and operation of a public service and they therefore both consist of output specification systems. Further, both contracts can be considered to be fixed-price contracts to the extent that the remuneration of the private provider does not depend on his costs. In other words, under both types of contract

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2Iossa and Martimort (2008) distinguished between three types of PPP contract, depending on whether payment is based on (i) user charges, (ii) usage or (iii) availability. In the first case, the private provider bears all the demand risk. In the second, the allocation of demand risk depends on the relationship between payment and actual level of usage. In the third case, the public authority retains all demand risk. Although it is contractually possible to restrict the degree of demand risk imposed on the private provider (Athias and Saussier, 2007), so that public authorities do not face a binary choice of contracts, this does not alter the results obtained herein. This is because the lesser the extent to which the private provider bears demand risk, the weaker his probability of bankruptcy, all else being equal.
the procuring authority offers the private provider a pre-specified price to complete the project. Moreover, these contracting procedures do not imply different magnitudes of the burdens on the private operator, because both formally delegate sufficient residual control rights to him to provide a service free of interference. The main difference between these two contracts therefore concerns the allocation of demand risk. Section 3.1 presents a simple incomplete contract model of contractual design for the provision of a public service by a private provider.

3.1 Model framework

We consider a contractual relationship in which a public authority \( PA \) contracts with a private provider \( PM \), which generates a benefit \( \tilde{b} \) and a consumer surplus \( CS \). The private provider faces a fixed cost \( F \) for the provision of the service.\(^3\) He invests \( e \) in cost-reducing efforts \(^4\) (the cost of the investment is \( e \)), which generates for him a cost advantage of \( w(e) \).\(^5\) The investment \( e \) and the benefit \( w(e) \) are non-contractible. We assume that this cost-reducing investment is relationship-specific, i.e. \( PM \) gains no benefit from \( e \) outside this relationship.

The public authority makes a non-verifiable investment \( j \) (with cost \( j \)), which corresponds to an effort to adapt of the public service provision over time in order to meet changing consumer demand (i.e. demand-enhancing effort). We assume no interaction between adaptation efforts and cost-reducing efforts in line with Hart (1995). Because the private provider has control rights over the service provision, the public authority must renegotiate the contract with him in order to implement \( j \). However, \( j \) is not relationship-specific, i.e. the public authority can exploit \( j \) in the case of a renegotiation failure when the private provider goes bankrupt. Both the benefit \( \tilde{b} \) and consumer surplus \( CS \) are increasing functions of \( j \).

As highlighted, a critical aspect of any PPP is the allocation of demand risk between the public authority and private provider. In a mechanism based on availability, the public authority retains all the demand risk and benefits from the service. By contrast, in a payment mechanism based on demand level, it is the private provider who retains all the benefits. Note that the objective function of the public authority also includes the consumer surplus.

We assume that when the private provider bears the demand risk, he may get negative profits, leading to the failure of the concession and his exit from the market (in other words, he goes bankrupt).\(^6\) This is never the case for the public authority. We also assume that the private provider cannot be replaced by a new one except when he goes bankrupt. This assumption rests on the fact that the adaptation is extra-contractual and that the private provider cannot be legally

\(^{3}\)Variable costs are omitted because most PPPs relate to natural monopolistic activities.  

\(^{4}\)Because in both contractual designs \( PM \) has control rights over the service provision, \( e \) will be implemented unilaterally.  

\(^{5}\)Contrary to Hart, Shleifer and Vishny (1997), we do not consider that the cost-reducing efforts of the private provider adversely affect quality. In fact, although some previous studies have shown that the quality shading hypothesis is true for government services (e.g. Deber, 2002), others have found the opposite, namely that cost-reducing efforts are accompanied by an increase in quality (e.g. McDavid, 1985).  

\(^{6}\)The profitability of most PPP contracts is sensitive to demand; indeed a marginal change in demand is often enough to generate negative profits for the private provider. Guash (2004) reported, for instance, that approximately 6\% of the toll road concessions granted worldwide in 1990–2001 were eventually abandoned.
sanctioned for failing to implement it.\textsuperscript{7}

Demand for the service is denoted $D$. For simplicity, we consider that demand is sensitive to innovations when they are sufficiently important.\textsuperscript{8} In other words, we assume that the demand is $\varepsilon > 0$ when an innovation with a low value $j$ is implemented (e.g. a low “quality” innovation), $j < j_0$. It is a function of $j$, $Q(j)$ with $Q(j) > \varepsilon$ and $Q' > 0$, when an innovation with a high value is implemented, $j \geq j_0$.

Formally,

$$D(j) = \begin{cases} 
Q(j) & \text{if an innovation } j \geq j_0 \text{ is implemented} \\
\varepsilon, & \text{otherwise.}
\end{cases} \quad (1)$$

When the innovation effort is small, $j < j_0$, or no innovation is implemented, demand is $\varepsilon$ and it is randomly distributed over $[\varepsilon, \overline{\varepsilon}]$ with $0 < \varepsilon < \overline{\varepsilon}$. This assumption deserves further comments. Recall that the innovation can only be implemented with the approval of the private provider. Hence, we assume that demand is $Q(j)$ when an investment $j$ of sufficient magnitude ($j \geq j_0$) is made by the public authority and when this innovation is implemented by the private provider. Conversely, demand is $\varepsilon$ when an investment of insufficient magnitude ($j < j_0$) is made by the public authority or when the innovation is not implemented (whatever the magnitude of the investment $j$). However, to avoid the (uninteresting) case where equilibrium innovation efforts end up being the same regardless of the allocation of demand risk, we focus on the case where the threshold $j_0$ is sufficiently low and then $j^*,j_{CC},j_{AC} \geq j_0$ where $j^*$ denotes the first best level of innovation, $j_{CC}$ the equilibrium level of innovation when $PM$ bears the demand risk and $j_{AC}$ the equilibrium level of innovation when $PA$ bears the demand risk.

We normalize the price to unity. If there is no bankruptcy, the benefit is then given by

$$\tilde{b}(j) = D(j). \quad (2)$$

We denote $\hat{\varepsilon} = E[\varepsilon]$, where $E[\cdot]$ represents the expectations operator. We also use the following notations:

$$b(j) = Q(j) \text{ and } \tilde{b}(\varepsilon) = \varepsilon. \quad (3)$$

As mentioned above, the private provider is the only agent that can go bankrupt, which can occur only when he bears the demand risk. When he bears the demand risk and the innovation $j$ is not implemented, the payoff for the private provider is $\tilde{b}(\varepsilon) - F + w(e) - e$. Bankruptcy occurs when this payoff is negative and, consequently, the probability of bankruptcy is defined as:

$$\rho = \Pr(\varepsilon < -w(e) + e + F). \quad (4)$$

Hence, $\rho$ is a decreasing function of $w(e) - e$. Let us denote $\rho \equiv \rho(w(e) - e)$ with $\rho' < 0$. In

\textsuperscript{7} In practice, the public authority can break the contract but this is prohibitively expensive; for this reason we do not observe any contract breach in practice.

\textsuperscript{8} Iossa and Martimort (2008 and forthcoming) considered the same kind of simple demand function and assumed that demand is inelastic when the price is sufficiently small.
the following, we focus on interior solutions and make the usual assumptions about the functions $b$, $CS$, and $w$: $b(0) < +\infty$, $CS(0) = CS_0 = 0$, $b' + CS' > 0$, $b'' + CS'' < 0$, $\lim_{j \to 0} (b' + CS') > 1$, $\lim_{j \to +\infty} (b' + CS') < 1$; $w' > 0$, $w'' < 0$, $w(0) = 0$, $\lim_{e \to 0} w'(e) = +\infty$ and $\lim_{e \to +\infty} w'(e) = 0$.

The timing of the model is as follows:

**Stage 0**: Demand risk is either with $PA$ or with $PM$.

**Stage 1**: $PA$ and $PM$ sink their respective investments $j$ and $e$.

**Stage 2**: Renegotiation takes place to allow the adaptation to be implemented in the service provision; $PA$ and $PM$ share the surplus generated by the non-verifiable efforts à la Nash bargaining (with symmetric bargaining power).

**Stage 3**: $PA$ and $PM$ trade (jointly or with their market alternatives).

### 3.2 Default payoffs and renegotiation gain

Default payoffs are those received by each contracting party when innovation $j$ is not implemented. They are then determined without considering the sunk investments.

**When the private provider bears the demand risk**, he can go bankrupt (with probability $\rho$) if no innovation is implemented. We assume that this is the only situation in which the public authority can replace one private provider with another. Moreover, in this case, the public authority will use an incentive contract such that the new private provider will implement the innovation, and it is she that obtains all the benefits, $CS(j) + b(j)$. If the innovation is not implemented and the private provider does not go bankrupt (with a probability $1 - \rho$), the payoff of the public authority is the consumer surplus level associated with the basic service, $CS(0)$, normalized to zero. Therefore, when the private provider bears the demand risk, the expected default payoff of the public authority is $\rho(CS(j) + b(j))$.

The expected default payoff of the private provider is more complicated. When no innovation is implemented, the expected benefit of the private provider, conditional on the fact that he does not go bankrupt, is:

$$b_0(w(e) - e) \equiv E[\epsilon / \epsilon - F + w(e) - e \geq 0].$$

Hence, the expected default payoff of the private provider is given by:

$$E[b(\epsilon) + w(\epsilon) / b(\epsilon) - F + w(e) - e \geq 0] = (1 - \rho(e))(b_0(w(e) - e) + w(e)).$$

**When the public authority bears the demand risk**, the private provider cannot go bankrupt and the public authority cannot therefore replace him. The contract is for the availability of the basic service and the public authority therefore pays a fixed amount $t_0$, normalized to zero, to the private provider. Therefore, the default payoff of the private provider is $w(\epsilon)$, and the default payoff of the public authority is $CS(0) + E[\epsilon] = \hat{\epsilon}$.

Let us denote $e^*$ as the first best level of cost-reducing effort, $e_{CC}$ the equilibrium level of cost-reducing effort when $PM$ bears the demand risk and $e_{AC}$ the equilibrium level of cost-reducing effort when $PA$ bears the demand risk.
The gain from the renegotiation process is the difference between the total surplus when the renegotiation is successful (this is $CS+b+w$ and does not depend on the allocation of demand risk), and the total surplus when the negotiation fails (the sum of the default payoffs, which, by contrast, does depend on the allocation of demand risk). Following Grossman and Hart (1986), we assume that the solution of the renegotiation is the Nash bargaining solution (with equal bargaining power), namely the players share the negotiation gain equally.

3.3 First best

The first best solution is the vector of investments $(j^*, e^*)$ that maximizes the total surplus, namely the sum of consumer surplus, the benefits and the cost advantage net of the fixed cost and investment costs.

$$(j^*, e^*) = \arg \max_{(j,e)} \{CS(j) + b(j) - F + w(e) - j - e\},$$

(7)

The unique solution $(j^*, e^*)$ is such that:

$$CS'(j^*) + b'(j^*) = 1,$$

(8)

$$w'(e^*) = 1.$$  

(9)

At the social optimum, the marginal benefits of the investments must equal their marginal costs, which are 1 in both cases.

3.4 Equilibrium when the private provider bears demand risk

Suppose that the private provider bears the demand risk. The gain from the renegotiation process is the difference between the total surplus when the renegotiation is successful $CS+b+w$ and the sum of the default payoffs, which is $\rho (CS+b) + (1 - \rho) (b_0 + w)$ when the private provider bears the demand risk. The gain from the renegotiation for each side is then

$$\frac{1}{2} [CS + b + w - \rho (CS + b) - (1 - \rho) (b_0 + w)]$$

(10)

$$= \frac{1 - \rho}{2} [CS + b - b_0] + \frac{\rho}{2} w.$$  

The payoff of each agent is the sum of his or her default payoff and his or her gain from the renegotiation net of the cost of his or her individual investment. Formally, $PM$’s payoff is given by:

$$U_{PM} = (1 - \rho) (b_0 + w) + \frac{1 - \rho}{2} [CS + b - b_0] + \frac{\rho}{2} w - F - e$$

(11)

$$= \frac{1 - \rho}{2} (CS + b + b_0) + \left(1 - \frac{\rho}{2}\right) w - F - e,$$
and, PA’s payoff is:

\[
U_{PA} = \rho (CS + b) + \frac{1 - \rho}{2} [CS + b - b_0] + \frac{\rho}{2} w - j
\]

\[
= \frac{1 + \rho}{2} (CS + b) - \frac{1 - \rho}{2} b_0 + \frac{\rho}{2} w - j.
\]

(12)

Note that when the bankruptcy risk is null, namely \( \rho (e) \equiv 0 \), the parties split the gain from adaptation equally (there is no gain from the cost savings). \( PM \) chooses \( e \) that maximizes his payoff, \( U_{PM} \), and \( PA \) chooses \( j \) that maximizes her utility \( U_{PA} \). The solution is denoted \((e_{CC}, j_{CC})\) and solves:

\[
Max_{e \geq 0} \left\{ U_{PM} = \frac{1 - \rho((w(e) - e))}{2} (CS (j) + b(j) + b_0 ((w (e) - e))) \right. \\
\left. + \frac{1 - \rho((w(e) - e))}{2} w (e) - F - e \right\},
\]

(13)

and,

\[
Max_{j \geq 0} \left\{ U_{PA} = \frac{1 + \rho((w(e) - e))}{2} (CS (j) + b(j)) \right. \\
\left. - \frac{1 + \rho((w(e) - e))}{2} b_0 ((w (e) - e)) + \frac{\rho((w(e) - e))}{2} w (e) - j \right\}.
\]

(14)

The investments \( (e_{CC}, j_{CC}) \) are characterized by the following two first order conditions (we have omitted the arguments for simplicity):

\[
\frac{1}{2} \left[ -\rho' (CS + b + b_0 + w) + (1 - \rho) b_0' \right] (w' - 1) + \frac{1 - \rho}{2} w' = 1,
\]

(15)

Incentive effect

\[
\frac{1}{2} \left[ -\rho' (CS + b + b_0 + w) + (1 - \rho) b_0' \right] (w' - 1) + \frac{1 - \rho}{2} w' = 1,
\]

Hold-up effect

and,

\[
\frac{1 + \rho}{2} (CS' + b') = 1.
\]

(16)

Both investments deviate from the first best.

Firstly, the choice of cost-reducing effort, \( e_{CC} \), is driven by two effects, namely an incentive effect and a hold-up effect. The incentive effect pushes \( PM \) to choose an effort level close to the first best level. In fact, the term in brackets is always positive, \( \left[ -\rho' (CS + b + b_0 + w) + (1 - \rho) b_0' \right] > 0 \), and the second term, \( (w' - 1) \), is positive for \( e < e^* \) and negative for \( e^* < e \). The term \( -\rho' (CS + b + b_0 + w) > 0 \) corresponds to a prevention effect, while the term \( (1 - \rho) b_0' > 0 \) corresponds to an insurance effect (see Ehrlich and Becker 1972).\(^9\) In other words, when the net benefit of the cost reducing effort, \( w(e) - e \), increases (i.e. \( w' - 1 \) is positive), \( PM \)’s probability of bankruptcy decreases and his expected benefit increases if he does not go bankrupt in the case of renegotiation failure. In turn, the hold-up effect pushes \( PM \) to choose a cost-reducing effort less than that of the first best, because the private provider does not receive the full return of his cost-reducing investment \( (w(e)) \) to the extent that he can go bankrupt. In fact, the risk of bankruptcy creates a hold-up

\(^9\)Ehrlich and Becker (1972) stated that (self-)insurance designates an effort aimed at loss reduction, for a given probability of loss, while prevention relates to an effort aimed at reducing the probability of a given loss.
opportunity for PA regarding the surplus generated by the cost-reducing efforts of PM. Because PM can go bankrupt and lose \( w \), the gain from the renegotiation for each side is increased by half the amount PM would lose in the case of renegotiation failure and bankruptcy, namely half of \( \rho w(e) \). Therefore, the addition of these effects is such that PM under-invests in his cost-reducing efforts, \( e_{CC} \leq e^* \).

Secondly, PA does not receive the full return of her investment in adaptation \( j \). This is explained by the fact that she needs the agreement of PM to implement the innovation. However, PM’s hold up is reduced because he can go bankrupt if he refuses to implement PA’s innovation.

The social surplus when PM bears the demand risk is defined as

\[
W_{CC} = CS(j) + b(j) - F + w(e) - j - \epsilon.
\]

(17)

3.5 Equilibrium when the public authority bears the demand risk

Suppose now that it is the public authority that bears the demand risk. The gain from renegotiation is still the difference between the total surplus when the renegotiation is successful \( CS + b + w \), and the sum of the default payoffs, which is \( w(e) + \hat{\epsilon} \) when the public authority bears the demand risk. The gain from the renegotiation process for each side is then \( \frac{1}{2}(CS(j) + b(j) + w(e) - w(e) - \hat{\epsilon}) \).

We can now write the payoffs of the two agents.

PM’s payoff is:

\[
U_{PM} = w(e) + \frac{1}{2}(CS(j) + b(j) - \hat{\epsilon}) - e - F,
\]

(18)

and, PA’s payoff is:

\[
U_{PA} = \hat{\epsilon} + \frac{1}{2}(CS(j) + b(j) - \hat{\epsilon}) - j.
\]

(19)

Note that when the demand risk remains with PA, because the public authority and private provider cannot go bankrupt, the probability \( \rho \) plays no role. PM chooses \( e \) that maximizes his payoff, \( U_{PM} \), while PA chooses \( j \) that maximizes her utility, \( U_{PA} \). The solution is denoted \( (e_{AC}, j_{AC}) \) and solves:

\[
Max_{e \geq 0} \left\{ U_{PM} = w(e) + \frac{1}{2}(CS(j) + b(j) - \hat{\epsilon}) - e - F \right\},
\]

(20)

and,

\[
Max_{j \geq 0} \left\{ U_{PA} = \hat{\epsilon} + \frac{1}{2}(CS(j) + b(j) - \hat{\epsilon}) - j \right\}.
\]

(21)

The investments \( (e_{AC}, j_{AC}) \) are characterized by the following two first order conditions:

\[
w'(e_{AC}) = 1,
\]

(22)

and,

\[
\frac{1}{2} (CS'(j_{AC}) + b'(j_{AC})) = 1.
\]

(23)
Contrary to the case in which $PM$ bears the demand risk, here $PM$ receives the full return on his cost-reducing investment because he cannot go bankrupt. However, $PA$ cannot replace the private provider and still needs his agreement to implement the innovation. Note that if there is no likelihood of bankruptcy, namely $\rho(e) \equiv 0$, then we would have $b_0 \equiv 0$ and hence (13) would be equivalent to (20) and (14) would be equivalent to (21). This means that there is no difference between the equilibrium effort levels under the two contract types when there is no likelihood of bankruptcy.

The social surplus in the case in which $PA$ bears the demand risk is defined as

$$W_{AC} = CS(j_{AC}) + b(j_{AC}) - F + w(e_{AC}) - j_{AC} - e_{AC}. \quad (24)$$

### 3.6 The choice of contractual design

The optimal contractual design is the one that generates the highest total surplus. It is socially desirable that $PM$ rather than $PA$ bears the demand risk only if

$$W_{CC} \geq W_{AC}, \quad (25)$$

or,

$$CS(j_{CC}) + b(j_{CC}) + w(e_{CC}) - j_{CC} - e_{CC} \geq CS(j_{AC}) + b(j_{AC}) + w(e_{AC}) - j_{AC} - e_{AC}. \quad (26)$$

The determination of the socially preferred allocation of demand risk requires a comparison between the investments levels under both contractual designs, as shown in Section 4.

### 4 Analysis of investments and the choice of the contractual design

Firstly, when the contract is designed such that the public authority bears the demand risk, this creates one distortion compared with the first best case. The public authority places $\frac{1}{2}$ weight on the benefits of adaptation (see equation (21)) instead of 1 in the first best case (see equation (7)). Regarding the cost-reducing effort of the private provider, however, there is no distortion (i.e. the conditions (22) and (9) are identical). Proposition 1 therefore follows directly from the first order conditions:

**Proposition 1:** When the public authority bears the demand risk, investments in adaptation are sub-optimal, $j_{AC} < j^*$, but cost-reducing investments are optimal, $e_{AC} = e^*$.

(The proof lies in the reasoning above).

Note that the probability of bankruptcy $\rho$ plays no role in this result because the private provider can only go bankrupt when he bears demand risk. The deviation from the first best case occurs

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10 And also from our assumptions about the properties of the function $CS + b$. 

12
because of the ability of the parties to renegotiate, in line with the seminal work of Hart, Shleifer and Vishny (1997). The possibility of hold-up reduces the incentives to make the appropriate adaptation efforts. However, the adaptation effort in our model is made by the public authority, which is important in the context of the next result.

Regarding the second contractual design, where the private provider bears the demand risk, the eventuality that he goes bankrupt when he does not implement the adaptation required by the public authority induces two distortions compared with the first contract type. Firstly, the public authority places $\frac{1+\rho}{2}$ weight on the benefit of adaptation (see equation (14)) instead of $\frac{1}{2}$ in the other contract type (see equation (21)). Therefore, when the probability of bankruptcy is positive ($\rho > 0$), the former is larger than the latter, $\frac{1+\rho}{2} > \frac{1}{2}$. Nevertheless, this weight is still lower that shown in the first best case. Secondly, we have shown (in Section 3.4) that the cost-reducing effort when the private provider bears demand risk is lower than the first best level, $e_{CC} < e^*$. Proposition 2 thus follows directly from the corresponding first order conditions:11

**Proposition 2:** The investment by the public authority in adaptation is lower when she bears the demand risk than when she does not, $j_{AC} \leq j_{CC} \leq j^*$, while the cost-reducing effort of the private manager is lower when he bears the demand risk than when he does not, $e_{CC} \leq e_{AC} = e^*$ (with $j_{AC} < j_{CC}$ and $e_{CC} < e_{AC}$ unless $\rho = 0$ and $j_{CC} < j^*$ unless $\rho = 1$).

(The proof lies in the reasoning above).

When the private provider bears the demand risk, he faces the risk of bankruptcy in the case of renegotiation failure. The public authority thus holds up a larger share of the benefits of the cost-reducing efforts, which incites the private provider to decrease his investment in cost-reducing efforts. The private provider in turn holds-up a smaller share of the adaptation benefits, which generates an incentive for the public authority to make a larger investment in adaptation. In other words, the imposition of demand risk, and hence bankruptcy risk, on the private provider increases the opportunities for the public authority to hold up him and reduces those for the private provider to hold up the public authority.

These results enable us to predict the choice of contractual design in Proposition 3:

**Proposition 3:**

1. Suppose that the function $CS(j) + b(j)$ is replaced by $\tau(CS(j) + b(j))$. Then, if $\tau$ is sufficiently small, it is preferable that the public authority rather than the private provider bears the demand risk.

2. Suppose that the function $w(e)$ is replaced by $\sigma w(e)$. Then, if $\sigma$ is sufficiently small, it is preferable that the private provider rather than the public authority bears the demand risk.

(The proof is given in the Appendix).

Part (1) of Proposition 3 assumes that $\tau$ tends to zero. The adaptation efforts $j_{CC}$, $j_{AC}$ and $j^*$ all tend to zero, at which point it is only the value of the cost-reducing effort that matters.

11 And from our assumptions on the functions $CS + b$ and $w$. 

13
Because the contract type under which the demand risk rests with the public authority is socially preferable regarding the cost-reducing effort, it is also preferable when $\tau$ to be sufficiently small. Part (2) of Proposition 3 is a symmetrical result. This part assumes that as $\sigma$ approaches zero, the cost-reducing efforts $e_{CC}$, $e_{AC}$ and $e^*$ all approach zero, too. Then, it is only the value of the adaptation effort that matters. Because the contract type under which the demand risk rests with the private provider is socially preferable from the point of view of adaptation, it is also preferable when $\sigma$ to be sufficiently small.

Proposition 3 highlights two main aspects: (i) that no contractual design is optimal and (ii) that no contractual design is always preferable. In other words, the type of contract in which the demand risk rests with the private provider always dominates that in which the demand risk rests with the public authority, in terms of the incentives for the latter to invest in demand-enhancing efforts. However, the contract in which the demand risk rests with the public authority always dominates that in which the demand risk rests with the private provider in terms of cost-reducing incentives for the private provider. A trade-off therefore occurs between adaptation and cost-reducing efforts in the allocation of demand risk.

We conclude that when the benefits of adaptation are important, it is socially preferable to design a contract in which the demand risk rests with the private provider, whereas when the benefits of cost-reducing efforts are important, it is socially preferable to place the demand risk on the public authority.

5 Case Studies

This section uses two well-known case studies to illustrate the underlying logic of the model presented in the previous sections. The first case study from the UK illustrates the case of a contract in which the demand risk rests with the public authority (the school catering case), whereas the second, from France, portrays the case of a contract in which the demand risk rests with the private provider (the highway case).

5.1 School catering case

Let us first consider the British school catering case first mentioned in Section 2.2. In essence, the British government pledged to rid school menus of ‘junk’ food after a series of television reports on the poor quality of school dinners by celebrity chef Jamie Oliver in early 2005. However, new schools locked into 25-year contracts through private finance initiatives (PFIs) found that they could not alter their menus despite the pledge of the government.

Because the public authority typically retains the demand risk under PFI contracts, the private provider, who does not bear the demand risk, invests in cost-reducing efforts, while the procuring authority has very limited powers to make him adapt the service according to the change required by the government according to the demand for healthy food from the public. This perfectly illustrates Proposition 2: that there is weak adaptation under contracts in which the public authority bears
the demand risk, whereas the cost-reducing efforts of the private provider are significant.

Now consider the characteristic of this case in light of our theoretical predictions regarding the choice of contractual design. The social gain of high-quality school catering is high owing to public health considerations, because junk food is now considered to be a primary cause of disease. Further, the cost of not providing school dinners is potentially high in terms of (i) the opportunity costs for parents if they decide to cook lunch for their children everyday and (ii) security if they allow them to buy lunch for themselves. If we follow Proposition 3 (1), this means that the value of \( \tau \) is high, implying that it is preferable for the private provider, rather than the public authority, to bear the demand risk. If such a choice is made, our model predicts that adaptation would be more likely.

In the case of universities, however, placing the demand risk on a private provider is less likely to be socially preferable. This is because, the considerations of the healthy consequences of junk food, security and the opportunity costs for parents would be relatively less important, implying that the benefits of adaptation would be fewer.

5.2 Highway case

Let us now consider the case of the “Shipwrecked Men of the Road of Saint-Arnoult-In-Yvelines”. After unexpectedly heavy snowfall in 2003, the French government pressured the private provider to adapt his service provision according to consumer demand. Further, the private provider accepted his responsibility to invest in lighter gritting vehicles and automatic gritting systems.

It is important to note at this stage that in France the maintenance of highways is agreed through contracts in which the demand risk rests with the private provider. Thus, in contrast to the UK case, this case study highlights that when the private provider bears demand risk, service adaptation can occur following changing public demand in line with Proposition 2 (\( j_{CC} > j_{AC} \)).

However, we speculate that the contractual design that is socially preferable is not to make the private provider bear the demand risk. In the case of highways, we expect the potential benefits from non-contractible cost-reducing efforts to be high, whereas the potential benefits from non-contractible adaptation efforts are low (because the uncertainty regarding consumer preferences over time is weak). Accordingly, the socially preferable contract design places the demand risk on the public authority (in line with Proposition 3(1)).

These results are generally consistent with existing evidence on how PFIs - and thus contracts in which demand risk rests with the public authority- work. According to a report commissioned by the Treasury Taskforce (Arthur and Andersen and Enterprise LSE 2000), PFIs generate substantial cost savings when applied to contracts for roads, although they work less well for those related to schools and hospitals.
6 Conclusion

The present paper has investigated how demand risk allocation influences the incentives of the contracting parties in PPPs. It focused on the adaptation investments of the public authority and voluntarily omitted the adaptation efforts of the private provider. Although the findings show that the consideration of the adaptation efforts of both parties does not affect the results if the adaptation benefits are separable, an interesting extension in future research might be to consider their interactions (i.e. whether they are complementary or substitutable). Although this might introduce further complexity into the analysis, it could lead to interesting insights. Another interesting extension in future research would be to consider that the public authority could also go bankrupt; however, this could be not trivial because the roles of the private provider and public authority are dissimilar.

Although our model is simple, it captures the most important trade-offs at stake when considering the efficiency of PPPs. We first showed that the incentives of the contracting parties weaken when they bear demand risk, which we described as a new case of counter-incentives. We also showed that demand risk allocation varies according to the relative importance of the benefits of adaptation compared with the benefits of cost-reducing efforts, highlighting the fact that no contractual design is optimal or always dominant. In addition, this paper suggests that the current trend towards the increasing adoption of contracts in which demand risk rests with the public authority should not necessarily lead to the abolition of contracts in which demand risk rests with the private provider, but rather that the two contract types should coexist.

Appendix

Proof of Proposition 3: To show (1), replace \( CS(j) + b(j) \) by \( \tau (CS(j) + b(j)) \). The conditions (16) and (23) become:

\[
\frac{1 + \rho}{2} (CS'(j_{CC}) + b'(j_{CC})) = \frac{1}{\tau},
\]

and,

\[
\frac{1}{2} (CS'(j_{AC}) + b'(j_{AC})) = \frac{1}{\tau}.
\]

Since \( CS'' + b'' < 0 \) and \( \lim_{j \to 0} (b' + CS') = +\infty \), when \( \tau \) approaches 0, both \( j_{CC} \) and \( j_{AC} \) also approach 0. Thus, the social surplus when \( PM \) bears demand risk becomes:

\[
\lim_{\tau \to 0} W_{CC} = w(e_{CC}) - F - e_{CC},
\]

and the social surplus in the case where \( PA \) bears demand risk becomes:

\[
\lim_{\tau \to 0} W_{AC} = w(e_{AC}) - F - e_{AC}.
\]
According to Proposition 1, we have $e_{AC} = e^*$ where $e^*$ maximizes $e \mapsto w(e) - e$. Using Proposition 2, we know that $e_{CC} \leq e_{AC}$ (with $e_{CC} < e_{AC}$ unless $\rho = 0$) and then,

$$\lim_{\tau \to 0} W_{CC} \leq \lim_{\tau \to 0} W_{AC},$$

(31)

with $\lim_{\tau \to 0} W_{CC} < \lim_{\tau \to 0} W_{AC}$ unless $\rho = 0$.

To show (2), replace $w(e)$ by $\sigma w(e)$. The conditions (15) and (22) become:

$$1 - \frac{\rho}{2} w'(e_{CC}) = \frac{1}{\sigma},$$

(32)

and,

$$w'(e_{AC}) = \frac{1}{\sigma}.$$  

(33)

Since $w'' < 0$ and $\lim_{e \to +\infty} w'(e) = 0$, when $\sigma$ approaches 0, both $e_{CC}$ and $e_{AC}$ also approach 0. Thus, the social surplus when $PM$ bears demand risk becomes:

$$\lim_{\tau \to 0} W_{CC} = CS(j_{CC}) + b(j_{CC}) - F - j_{CC},$$

(34)

and the social surplus when $PA$ bears demand risk becomes:

$$\lim_{\tau \to 0} W_{AC} = CS(j_{AC}) + b(j_{AC}) - F - j_{AC}.$$  

(35)

According to Proposition 2, the investment in adaptation is closer to the (unique) optimal value for a concession contract compared with an availability contract, $j_{AC} \leq j_{CC} \leq j^*$ (with $j_{AC} < j_{CC}$ unless $\rho = 0$) and then $\lim_{\tau \to 0} W_{AC} \leq \lim_{\tau \to 0} W_{CC}$ (with $\lim_{\tau \to 0} W_{AC} < \lim_{\tau \to 0} W_{CC}$ unless $\rho = 0$).

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