« Fuzzy risk adjusted performance measures: application to Hedge funds »

Alfred MBAIRADJIM MOUSSA
Jules SADEFO KAMDEM
Michel TERRAZA

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Fuzzy risk adjusted performance measures: application to Hedge funds

A. Mbairadjim Moussa
LAMETA
Université Montpellier I

J. Sadefo Kamdem*
LAMETA-CNRS
Université Montpellier I

M. Terraza
LAMETA-CNRS
Université Montpellier I

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Abstract

In this paper, following the notion of probabilistic risk adjusted performance measures; we introduce that of fuzzy risk adjusted measures (FRAM). In order to deal efficiently with the closing-based returns bias induced by market microstructure noise, as well as to handle their uncertain variability, we combine fuzzy set theory and probability theory. The returns are first represented as fuzzy random variables and then used in defining fuzzy versions of some adjusted performance measures. Using a recent ordering method for fuzzy numbers, we propose a ranking of funds based on these fuzzy performance measures. Finally, empirical studies carried out on fifty French Hedge Funds confirm the effectiveness and give the benefits of our approach over the classical performance ratios.

Keywords: Asset allocation; Fuzzy sets theory; Fuzzy random variables; Hedge Funds; Performance measures.

*Corresponding author. LAMETA Université de Montpellier I, UFR d’Economie Avenue Raymond DUGRAND - Site de Richter C.S. 79606 34960 MONTPELLIER CEDEX 2 France, Courriel: mbairadjim@lameta.univ-montp1.fr, sadefo@lameta.univ-montp1.fr
1 Introduction

A hedge fund can be defined as a "pooled investment vehicle that is privately organized, administered by professional investment managers, and not widely available to the public"\textsuperscript{1}. Due to their private nature and because they are not subjected to several requirements of regulatory bodies, there is a lack of transparency of the hedge fund managers' activities. As hedge fund managers have the possibility to not disclose their performance, daily published results are often subject to bias (Eling (2006)). The use of such biased data for the systematic risk (beta) estimation tends to lead to inconsistent ordinary least squares estimators in linear pricing models such as Capital Asset Pricing Model (CAPM) or the Arbitrage Pricing Theory (APT)\textsuperscript{2}. Generally speaking, for linear regression models with measurement errors in the regressors estimated by ordinary least square method, Cragg (1994) demonstrated that the slope coefficients were biased toward zero and concluded that the measurement error "produced a bias of the opposite sign on the intercept coefficient when the average value of the explanatory variables is positive". It follows that the presence of noise in the return biases the estimates of the systematic risk beta and of the Jensen’s alpha, leading to the prominence of the performance evaluation based on the linear factor market models.

Moreover, the assumption of linearity of the causal relationship of returns with a set of covariates usually referred to as risk factors as well as that of the normal-distribution of financial assets returns formulated by seminal researchers (Markowitz, Sharpe, Treynor,...) in quantitative finance, have been extensively discussed in the literature in recent years. For the special case of hedge funds, these two assumptions are widely violated as shown in Agarwal and Naik (2001), Mitchell and Pulvino (2001) and empirically confirmed by Amin and Kat (2003). The violation of these two assumptions implies invalidity of the CAPM for hedge funds performance evaluation. Hence the use of the traditional adjusted performance measures becomes questionable. This conclusion and others similar ones have motivated some authors such as Cappoci and Hübner (2004), Coen and Hübner (2009), Darolles and Gourrieroux (2010) to propose alternatives to the adjusted performance measures derived from the Sharpe’s market line based on probability theory.

In this paper, we focus on hedge funds performance evaluation. We propose combining fuzzy set theory and probability theory to construct some adjusted performance measures. Our modeling approach aims to deal with imprecision induced by market microstructure noise and the stochastic variability of the risk factors. As explained by Shapiro (2009), these two sources of the uncertainty can both be modeled by a fuzzy random variable. For this purpose, the basic assumption of our modeling approach is the representation of financial assets returns through a fuzzy random variable. Note that the fuzzy representation of financial asset return has been resorted in literature by many authors including, among others, Tanaka and Guo (1999), Smimou et al. (2008) and Yoshida (2009). For an overview of some applications of fuzzy logic in insurance, see Shapiro (2004) and some references therein.

This paper studies the fuzziness of returns over a period, as the effect of noise induced by the market microstructure frictions on the observed returns. Our fuzzy set-valued returns can be seen as a generalized form of the interval-valued and the real-valued one. This approach aims at determining adjusted performance measures by taking into account the imprecision of the risk factors. We will first focus on the estimation of the market line by considering the returns as fuzzy random variables.

\textsuperscript{1}A definition adapted from Amin and Kat (2003)

\textsuperscript{2}Klepper and Leamer (1984), Leamer (1987), among others, provide evidence of inconsistency of ordinary least square estimators in linear regression models with measurement errors in the regressors.
This question was the subject of one of our recent reflections which will be referred to as Mbairadjim et al. (2012) hereafter.

The remainder of this paper is organized as follows. The section 2 is a brief presentation of basic concepts of fuzzy set theory necessary to the introduction of the fuzzy presentation process of monthly returns in section 3. Section 4 is devoted to the definition and mathematical characterization of fuzzy adjusted risk measures. In Section 5, we review an ordering method for fuzzy number which can be applied in order to produce FRAM based funds ranking. An application to hedge funds data from France is given in Section 6. We determine and compare the rankings associated with the classical crisp performance measures and the fuzzy adjusted performance measures. Finally some conclusions are listed in Section 7.

2 Preliminaries
Before proceeding to formal presentation of fuzzy adjusted performance measures, we first briefly review three of the basic concepts of fuzzy theory; namely fuzzy sets, fuzzy numbers and fuzzy random variables. Readers familiar with these topics can skip this section, and those interested in a detailed presentation of fuzzy theory, may see Zimmermann(2001).

2.1 Fuzzy sets and fuzzy numbers
Let \( X \) a crisp set whose elements are denoted \( x \). A fuzzy subset \( A \) of \( X \) is defined by its membership function \( \mu_A : X \to [0, 1] \) which associates each element \( x \) of \( X \) with its membership degree \( \mu_A(x) \) (Zadeh (1965)). The degree of membership of an element \( x \) to a fuzzy set \( A \) is equal to 0 (respectively 1) if we want to express with certainty that \( x \) does not belong (respectively belongs) to \( A \).

The crisp set of elements that belong to the fuzzy set \( A \) at least to the degree \( \alpha \) is called the \( \alpha \)-cut or \( \alpha \)-level set and defined by:

\[
A_\alpha = \{ x \in X | \mu_A(x) \geq \alpha \}
\]

\( A_\alpha \) is the closure\(^3\) of the support \(^4\) of \( A \).

Fuzzy numbers are numbers that have fuzzy properties, examples of which are the notions of "around ten percent" and "extremely low". Dubois and Prade (1980, p.26) characterizes the fuzzy numbers as follows

**Definition 2.1** A fuzzy subset \( A \) of \( \mathbb{R} \) with membership \( \mu_A : \mathbb{R} \to [0, 1] \) is called fuzzy number if

1. \( A \) is normal, i.e. \( \exists x_0 \in \mathbb{R} | \mu_A(x_0) = 1 \);

2. \( A \) is fuzzy convex, i.e.

\[
\forall x_1, x_2 \in \mathbb{R} \ | \ \mu_A(\lambda x_1 + (1 - \lambda)x_2) \geq \min\{\mu_A(x_1), \mu_A(x_2)\}, \ \forall \lambda \in [0, 1];
\]

3. \( \mu_A \) is upper semi continuous\(^5\);

\(^3\)The closure of the support of \( A \) is the smallest closed interval containing the support of \( A \) (Shapiro (2009))

\(^4\)The support of \( A \) is the set of all \( x \) such that \( \mu_A(x) > 0 \). (Shapiro (2009))

\(^5\)Semi-continuity is a weak form of continuity. Intuitively, a function \( f \) is called upper semi-continuous at point \( x_0 \) if the function’s values for arguments near \( x_0 \) are either close to \( f(x_0) \) or less than \( f(x_0) \)
4. supp(\(A\)) is bounded.

**Definition 2.2** (Zimmermann(1996, p. 64)) A LR-fuzzy number, denoted by \(\tilde{A} = (l, c, r)_{LR}\), where \(c \in \mathbb{R}^+\) is called central value, and \(l \in \mathbb{R}^+\) and \(r \in \mathbb{R}^+\) is the left and the right spread, respectively, is characterized by a membership function of the form

\[
\mu_{\tilde{A}}(x) = \begin{cases} 
L\left(\frac{x-c}{l}\right) & \text{if } c - l \leq x \leq c, \\
R\left(\frac{x-c}{r}\right) & \text{if } r + c \geq x \geq c, \\
0 & \text{else}.
\end{cases}
\]  

\(L: \mathbb{R}^+ \to [0, 1], R: \mathbb{R}^+ \to [0, 1]\) are strictly continuous decreasing functions such that \(L(0) = R(0) = 1\) and \(L(1) = R(1) = 0\). \(L\) and \(R\) are called the left and the right shape functions respectively. If right and left spreads are equal and \(L := R\), the LR-fuzzy number is said to be a symmetric fuzzy number and denoted \(\tilde{A} = (c, \Delta)\). \(\Delta\) is the spread equal to \(l = r\).

For simplicity, we limit the present study to triangular\(^6\) fuzzy numbers characterized by the shape function \(R(x) := L(x) := \max\{1 - x, 0\}\). The analysis can be extended to other membership types.

Using Zadeh’s extension principle (Zadeh (1965)), which is a rule providing a general method to extend a function \(f: \mathbb{R}^k \to \mathbb{R}\) to the set of fuzzy numbers, we can define binary operator such as addition, subtraction, multiplication... for two fuzzy numbers. When \(k = 2\), this method defines the membership function of the result as follows

\[
\mu_{\tilde{A}_1 \circ \tilde{A}_2}(z) = \sup_{(x_1, x_2) \in \tilde{A}_1 \times \tilde{A}_2} \{\min(\mu_{\tilde{A}_1}(x_1), \mu_{\tilde{A}_2}(x_2)) \mid x_1 \circ x_2 = z\}
\]  

where \(\circ\) is the binary operator.

### 2.2 Fuzzy random variables

Different approaches of the concept of fuzzy random variables have been developed in the literature since the 70’s. The most often cited being introduced by Kwakernaak (1978) and enhanced by Kruse and Meyer (1987), and the one by Puri and Ralescu (1986). An extensive discussion on these two approaches is given by Shapiro (2009). For the purpose of this study, we adopt the concept of FRVs of Puri and Ralescu (1986).

Let \(\mathcal{F}_c(\mathbb{R})\) denote the set of all normal convex fuzzy subsets\(^7\) of \(\mathbb{R}\) and \((\Omega, \mathcal{A}, P)\)\(^8\) a probability space.

More precisely, Puri and Ralescu (1986) have defined a FRV as follows

**Definition 2.3** The mapping \(X : \Omega \to \mathcal{F}_c(\mathbb{R})\) is said to be a FRV on \(\mathbb{R}\) if for any \(\alpha \in [0, 1]\), the \(\alpha\)-cut is a convex compact random set\(^9\).

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\(^6\)This assumption of simplicity is also made in numerous articles of IME such as Koissi and Shapiro(2006), Andrés-Sánchez (2007) and Berry-Stolze et al. (2010), among others.

\(^7\) A fuzzy set \(\tilde{A}\) is called a normal convex fuzzy subset of \(\mathbb{R}\) if \(\tilde{A}\) is normal, the \(\alpha\)-cuts of \(\tilde{A}\) are convex and compact and the support of \(\tilde{A}\) is compact. (Körner(1997))

\(^8\) Where \(\Omega\) is the set of all possible outcomes described by the probability space, \(\mathcal{A}\) is \(\sigma\)-fields of subsets of \(\Omega\), and the function \(P\) defined on \(\mathcal{A}\) is a probability measure.

\(^9\) A convex compact random set is a Borel-measurable mappings with the Borel \(\sigma\)-field generated by the topology associated with the Hausdorff metric on \(\mathcal{F}_c(\mathbb{R})\). (Gil et al. (2006))
Puri and Ralescu (1986) brought an expectation operator called the Aumann-Expectation and denoted \( E^A \) for FRVs. Its construction is based on the Aumann’s (1965) study on integrals of interval-valued functions. For a symmetric LR-fuzzy random variable \( \tilde{A} = \langle a, \Delta \rangle \), the Aumann-expectation \( E^A \) is defined by (Körner (1997)):

\[
E^A[\tilde{A}] = \langle E[a], E[\Delta] \rangle
\]  

(4)

Definition 4, shows that the Aumann-expectation is a linear operator as the expectation operator for real random variables.

Following the Puri and Ralescu’s approach of a FRV and based on the definition 4, Körner (1997) introduced a real-valued variance characterized by the Fréchet principle \(^{10}\), and covariance. For LR-fuzzy numbers \( \tilde{A}_i = \langle a_i, \Delta_i \rangle, \ i = 1, 2 \), Körner (1997) describes the variance and covariance operators as follow:

\[
\text{Cov}^A[\tilde{A}_1, \tilde{A}_2] = \text{Cov}[a_1, a_2] + l \text{Cov}[\Delta_1, \Delta_2],
\]

\[
\text{Var}^A[\tilde{A}] = \text{Var}[a] + l \text{Var}[\Delta],
\]

where

\[
l = \int_0^1 (L^{-1}(\alpha))^2 d(\alpha).
\]

\( l = \frac{1}{3} \) in the case of symmetric fuzzy triangular numbers

3 Fuzzy representation of returns

Since the introduction of FRVs were introduced as well-formalized models for fuzzy set-valued random elements, numerous studies in probability theory have been developed to analyze the properties of this new class of random variables. For the last three decades, one can mention those related to the formalization of measurability, to the laws of large numbers which strengthens the suitability of the fuzzy mean and to hypothesis testing. An overview of these developments on FRVs is available in Gil et al. (2006). Despite the existence of this complete mathematical analysis framework, the application of these theoretical results is still quite limited because of the difficulties encountered in the measurement and observation of FRVs in practice. Hence the necessity of building methods to provide fuzzy representations of observations, which are often crisp. A recent probabilistic solution was proposed by Gonzalez-Rodriguez et al. (2006) by the introduction of a family of fuzzy representation of random variables. Each of the representations transformed a crisp random variable into a FVR whose mean captured different relevant information on the probabilistic distribution of the original real-valued random variable. However the application of this method requires \textit{a priori} assumptions about the distribution of the real variable and about the shape of the membership function of the fuzzy random variable. This double assumption may lead to significant bias of information. There also exists other seminal ways to characterize fuzziness of a fuzzy variable crisply observed. Their review is given in

\(^{10}\) Fréchet (1948) the expectation operator \( \mathbb{E}^{(d)}[Z] \) and the variance \( \text{Var}^{(d)}[Z] \) for a random variable \( Z \) in a metric space \((M, d)\) as

\[
\mathbb{E}[d^2(Z, \mathbb{E}^{(d)} Z)] = \inf_{x \in M} \mathbb{E}[d^2(Z, x)],
\]

\[
\text{Var}^{(d)}[Z] = \mathbb{E}[d^2(Z, \mathbb{E}^{(d)} Z)].
\]
Dubois and Prade (1980, pp.255-64). This characterization generally consists in the estimation of the membership function. As pointed out by Ross (1995, pp.179-180), the assignment of the membership function can be intuitive or based on algorithms or logical operations. An example of such membership function assignment for a financial risk factor is given by Smimou et al. (2008). The asset return is represented by a fuzzy set when the investors face a situation in which the returns are vague or imprecise. The support of this fuzzy set determinates an inspiration interval in which the true value of return is located. The bid-ask spread is used as a proxy measure of the fuzziness whereas the crisp observed value of the return is assumed to be the central value of the fuzzy set. The authors used the bid-ask spread as fuzziness measure under the condition that it reflects the experts’ judgments. In addition, Kossi and Shapiro (2006, p.291) specified that a crisp data can be fuzzified by adding a number ±∆ to each value, where ∆ is chosen small compared to the center value. Following these two studies, we fuzzify the return of a hedge fund in order to reflect the effect of the market microstructure frictions on it observed value. For this purpose, we add positive numbers to the observed returns. For each period t, the number ∆ is chosen as the statistic summary of the noise during the period over this period. We proceed as follows

We denote by t the time period [t, t+1]. We partition price time series in sub-groups \( P_t = \{ P_{t+i/M}, i = 0, ..., M - 1 \} \) with size M each one corresponding to a period t. We adopt the most common scenario in the literature on the microstructure noise of the observed price process involving

\[
\log(P_{t+i/M}) = \log(P_{t+i/M}^*) + \epsilon_{t+i/M} \tag{5}
\]

where \( \epsilon_{t+i/M} \) is independent and identically distributed (i.i.d.) noise, independent of the frictionless price \( P_{t+i/M}^* \).

If only \( P_{t+i/M} \) is observable but not \( P_{t+i/M}^* \), the observed return over the sub-period \([i, i+1]\) in the period t, is given by,

\[
r_{t+i/M} = \log(P_{t+(i+1)/M}) - \log(P_{t+i/M}), \quad i = 0, ..., M - 1. \tag{6}
\]

The observed return so defined, is noise contaminated. It is linked to the frictionless return \( r_{t+i/M}^* = \log(P_{t+(i+1)/M}^*) - \log(P_{t+i/M}^*) \) as,

\[
r_{t+i/M} = r_{t+i/M}^* + \nu_{t+i/M} \tag{7}
\]

where

\[
\nu_{t+i/M} = \epsilon_{t+(i+1)/M} - \epsilon_{t+i/M} \tag{8}
\]

with \( i = 0, ..., M - 1 \) and \( t = 1, ..., T \).

At period t, the noise contained in the observed return is the random variable \( \nu_t \). Since \( \epsilon_{t+i/M} \) and \( \nu_{t+i/M} \) are zero mean for \( i = 0, ..., M - 1 \), \( \nu_t \) is also zero mean. Then, based on the observed

\[
R_t = \log \left( \frac{P_{t+1}}{P_t} \right) = \sum_{i=0}^{M-1} \log \left( \frac{P_{t+(i+1)/M}}{P_{t+i/M}} \right) = \sum_{i=0}^{M-1} r_{t+i/M} = \sum_{i=0}^{M-1} r_{t+i/M}^* + \sum_{i=0}^{M-1} \nu_{t+i/M}. \tag{9}
\]
information, a natural range of the possible values of $\nu_t$ could be given by $[-\sigma_{\nu,t}, \sigma_{\nu,t}]$, where $\sigma_{\nu,t}$ is the standard deviation of the random variable $\nu_t$ within the period $t$ over which the return is calculated. It follows that an equivalent range of values of the frictionless return $R^*_t$ over the period $t$, is $[R_t - \sigma_{\nu,t}, R_t + \sigma_{\nu,t}]$, where $R_t$ is the observed return.

Under the assumptions that the random shocks $\nu_{t+i/M}$ are independent and identically distributed, mean zero with a bounded eighth moments and independent of the frictionless returns, Bandi and Russel (2006) shown that the second moment of the noise return within the period, is consistently estimated by the arithmetic average of the second powers of the return within the de period as follows

$$E[\nu^2_t] = \frac{1}{M} \sum_{i=0}^{M-1} \nu^2_{t+i/M}. \tag{11}$$

The standard deviation of $\nu_t$ is then consistently estimated by $\hat{\sigma}_{\nu,t} = \sqrt{E[\nu^2_t]}$.

We propose to fuzzify return over the period $t$ by adding $\pm \hat{\sigma}_{\nu,t}^{15}$ to the closing prices-based return. This fuzzy return is given by

$$\tilde{R}_t = \langle R^*_t, \Delta_t \rangle_L \tag{12}$$

where $R^*_t = \log \left( \frac{P_{t+1}}{P_t} \right)$ is the observed return over the period $t$ and $\Delta_t = \hat{\sigma}_{\nu,t}$

The fuzzy return is used to express that the frictionless return $R^*_t$ is around the observed and the imprecision is described by the spread$^{16} \Delta_t$. For example a stock which has a observed return equal to 0.15 over a period and 0.001 as the standard deviation of the noise returns within this period, the fuzzy return express that the efficient return is $0.15 \pm 0.001$. The membership to this interval, equals 1 for a return 0.15, and decreases with respect to the shape function $L$ on the right and left toward 0 for returns equal $0.15 + 0.001$ and $0.15 - 0.001$ respectively.

Finally, we can prove the following statement

**Proposition 1** The fuzzy return defined in (12) is a fuzzy random variable as given in Definition 2.3.

4 Fuzzy adjusted risk measures

The fuzzy representation of the monthly returns of a financial asset made in Section 3 is used in this section to formulate the fuzzy version of the performance ratios also called adjusted performance measures. These performance ratios introduced in 60s by Sharpe, Treynor, Jensen etc, result generally from the product of the risk premium of a financial asset over its risk measure. At equal returns, they

\[ \nu_t = \sum_{i=0}^{M-1} \nu_{t+i/M}, \tag{10} \]

It follows that

$hence \nu_t$ is mean zero.

$^{15}$The spread which is a measure of possible (not just probable) values of the return, is estimated by a probabilistic measure. However, the connection between probability and possibility is discussed by some authors such as Dubois et al. (2004) and Dubois (2006). More precisely, Dubois et al. (2004) justified the use of probabilistic quantities (quantiles) for determining the range of possible values via confidence intervals.

$^{16}$The spread term is used here in fuzzy set theory framework. It is not related to the conventional spread of a financial return.
define the most competitive financial asset the one that is the least risky. The performance ratios
can be presented in two groups depending on whether the investor’s risk measurement is absolute or
relative to a reference (benchmark). We present the fuzzy versions of two absolute ratios (Sharpe ratio
(Sharpe(1966)) and Treynor ratio (Treynor(1965))) and two relative ratios (Jensen’s alpha (Jensen
(1968)), information ratio (Black and Treynor(1973))). These performance measures derive from a di-
rect application of the theoretical results of the Capital Asset Pricing Model (Sharpe (1964), Lintner
(1965), Mossin (1966) and Treynor (1962) ). The definition of the CAPM with fuzzy returns is exten-
sively discussed in Mbairadjim et al. (2012). It is based on the assumptions of the original CAPM17
except that assumption of normal distribution of return is replaced by the assumption the returns are
LR-fuzzy random variables. The LR-fuzzy random return vector is denoted \( \tilde{R} = (\tilde{R}_1, ..., \tilde{R}_n) \) and the
riskless return is \( r_f \). Since the risk-free interest rate \( r_f \) is not subjected to uncertainty and known
with precision, it is defined as a real number despite of a fuzzy one.

For the preference of investors, we assume that each investor cares only about the Auman fuzzy
expected return \( \mathbb{E}^A[\tilde{R}_i] \) and its underlying standard deviation \( \sigma^A_i \) and all investors have the same
beliefs about investment opportunities: \( \mathbb{E}^A[\tilde{R}_i], \sigma^A_i \) and all correlations \( \sigma^A_{ij} \) for the \( n \) risky assets
(i, j = 1, ..., n).

Let \( \tilde{R}_i \) and \( \tilde{R}_B \) be the fuzzy returns of the asset \( i \) and of the benchmark \( B \). Under the above-
mentioned assumptions, the fuzzy market line18 is the following fuzzy linear regression model
\[
\tilde{R}_{it} = \tilde{\alpha}_i + \beta_i \tilde{R}_{it} + \tilde{\epsilon}_{it}
\]
where \( \tilde{\alpha}_i \) is a fuzzy parameter, \( \beta_i \) a real parameter and \( \tilde{\epsilon}_{it} \) the error term.

Further details on the specification of the error term and on the estimation of the fuzzy market
lines are available in Näther (2001), Wunsche and Näther (2002) and Mbairadjim et al. (2012).

Throughout the remainder of the paper, \( \oplus_H \) denotes Hukuhara difference19 (Hukuhara (1967))
of fuzzy numbers. In case of non-existence of the Hukuhara difference, we use its Stefanini (2010)
generalized form20.

### 4.1 Fuzzy Sharpe ratio

We introduce the fuzzy version of the Sharpe ratio by analogy to the definition of Sharpe (1966).

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17These assumptions include that capital market is completely competitive and frictionless, capital market clearing,
riskless borrowing, and lending are allowed.

18The specification \( \tilde{R}_i = \tilde{\alpha}_i + \beta_i \tilde{R}_m + \tilde{\epsilon}_i \) for the fuzzy market is also possible. However, the product \( \beta_i \tilde{R}_m \) of two
LR-fuzzy numbers does not always provide a LR-fuzzy number (Oussalah and De Schutter (2003) ) and consequently
implies further difficulty in the implementation of the fuzzy least square method for the model estimation. We limit
the present study to the crisp beta and an extension to a fuzzy beta needs to be addressed in future research. Another
possible specification could be \( \tilde{R}_i = \tilde{\alpha}_i + \beta_i \tilde{R}_m + \tilde{\epsilon}_i \), however it requires a real-valued random return \( \tilde{R}_m \) of the market.

19For two LR-fuzzy numbers \( \tilde{A}_1 = (l_1, a_1, r_1) \) and \( \tilde{A}_2 = (l_2, a_2, r_2) \), the Hukuhara difference \( \tilde{A}_1 \oplus_H \tilde{A}_2 \) exists if
\( l_1 \geq l_2, r_1 \geq r_2 \) and is given by \( \tilde{A}_1 \oplus_H \tilde{A}_2 = (l_1 - l_2, a_1 - a_2, r_1 - r_2) \).

20Stefanini (2010) introduced a generalization of the Hukuhara difference based on well-known standard interval
arithmetic by using the compact and convex \( \alpha \)-cuts of the fuzzy numbers. For two fuzzy numbers \( \tilde{A} \) and \( \tilde{B} \) such that
the \( \alpha \)-cuts are denoted by \( \tilde{A} = [\tilde{A}_a, \tilde{A}_b] \) and \( \tilde{B} = [\tilde{B}_a, \tilde{B}_b] \), the generalized Hukuhara difference \( \tilde{C} = \tilde{A} \oplus_H \tilde{B} \) is
classified by the \( \alpha \)-cuts \( \tilde{C}_a = \min\{\tilde{A}_a - \tilde{B}_a, \tilde{A}_a - \tilde{B}_b\}, \max\{\tilde{A}_a - \tilde{B}_a, \tilde{A}_b - \tilde{B}_b\} \). (Näther (2006, pp. 251)
Definition 4.1 The fuzzy Sharpe ratio of the asset $i$ is the fuzzy set $\tilde{S}_i$ defined by

$$\tilde{S}_i = \frac{E[A_t] \odot_H r_f}{\sqrt{Var^A[R_i]}}$$

(14)

The Propositions 4.2 gives the membership function of the fuzzy Sharpe ratio as follows

Proposition 4.2 Let $\tilde{R}_i$ be the fuzzy return of the asset $i$. If $\tilde{R}_i$ is a LR-fuzzy number with central $R_i$, left and right spreads $l_i$ and $r_i$ then the fuzzy Sharpe ratio is the LR-fuzzy number defined by

$$\tilde{S}_i = \left( \frac{E[l_i]}{\sqrt{Var^A[R_i]}}, \frac{E[R_i] - r_f}{\sqrt{Var^A[R_i]}}, \frac{E[r_i]}{\sqrt{Var^A[R_i]}} \right)_{LR}$$

(15)

4.2 Fuzzy Treynor ratio

Following the probabilistic definition of Treynor (1965), the fuzzy set-valued Treynor ratio is formally given as follows

Definition 4.3 Let $\beta_i$ be the real parameter of the fuzzy market line introduced in Eq. (13). The fuzzy Treynor ratio $\tilde{T}_i$ of the financial asset $i$ is the fuzzy number defined by

$$\tilde{T}_i = \frac{E[A_t] \odot_H r_f}{\beta_i}$$

(16)

Proposition 4.4 If the return of the asset $i$ is a LR-fuzzy number then the fuzzy Treynor ratio $\tilde{T}_i$ is the LR-fuzzy number defined by

1. If $\beta_i > 0$

$$\tilde{T}_i = \left( \frac{E[l_i]}{\beta_i}, \frac{E[R_i] - r_f}{\beta_i}, \frac{E[r_i]}{\beta_i} \right)_{LR}$$

(17)

2. If $\beta_i < 0$

$$\tilde{T}_i = \left( -\frac{E[r_i]}{\beta_i}, \frac{E[R_i] - r_f}{\beta_i}, -\frac{E[l_i]}{\beta_i} \right)_{LR}$$

(18)

4.3 Fuzzy Jensen’s alpha

Following the classical case (Jensen (1968)), the Jensen’s alpha derived from the fuzzy market line. It is the intercept of this fuzzy linear regression model.

Definition 4.5 The fuzzy Jensen’s alpha is the fuzzy number defined by

$$\tilde{\alpha}_i = \left( E^A[R_i] - r_f \right) \odot_H \beta_i \left( E^A[R_B] - r_f \right)$$

(19)

The fuzzy Jensen’s alpha so defined is a LR-fuzzy number because it is a linear combination of two LR-fuzzy numbers.
4.4 Fuzzy information ratio

By analogy to the classical approach of Treynor and Black (1973), the fuzzy information ratio measures a sub-performance of an asset relatively to a benchmark based on fuzzy returns.

**Definition 4.6** The fuzzy information ratio of an asset $i$ relatively to a benchmark $B$ is the fuzzy number defined by

$$\tilde{IR}_i = \frac{\mathbb{E}^A[\tilde{R}_i \ominus_H \tilde{R}_B]}{\sqrt{\text{Var}^A[\tilde{R}_i \ominus_H \tilde{R}_B]}}$$ (20)

The fuzzy information ratio above-defined, has a similar shape function as the fuzzy return since it is obtained by dividing the latter by a real number.

5 Fuzzy performance ratios and decision making

As in the classical case, ranking the fuzzy adjusted risk measures previously introduced is preliminary to decision making procedures. However, sorting fuzzy numbers remains a difficult problem in the literature of fuzzy set theory and its applications. Many solutions proposed over this last four decades sometimes led to unreasonable results (see Chen and Sanguansat (2011) and some references therein for details). The proposed ranking methods are generally based on the centroid index (Yager (1978)), defuzzification process on the coefficient of variation index ((chen (1996, 1998)) on the spreads and the defuzzified values of the fuzzy number (Chen and Chen (2007)). One of the recent methods based on the areas on the left and the right side of fuzzy number, proposed by Nejat and Mashinchi (2011) and holding the advantage of being easily implementable, is applied here for hedge funds fuzzy ratio-based ranking. Its adapted version for ranking symmetric triangular fuzzy numbers as in our case can be presented as follows

Let $A_1 = (l_1,c_1,r_1), \ldots, A_n = (l_n,c_n,r_n)$ be triangular fuzzy numbers with center $c_i$, left and right spreads $l_i, r_i$ respectively ($i = 1, \ldots, n$). Their membership functions denoted $f_1, \ldots, f_n$ respectively, can be expressed as:

$$f_i(x) = \begin{cases} 
\frac{x-c_i-l_i}{l_i} & \text{if } c_i-l_i \leq x \leq c_i, \\
\frac{c_i+r_i-x}{r_i} & \text{if } c_i+r_i \geq x \geq c_i, \\
0 & \text{else}.
\end{cases}$$ (21)

The inverse functions of $f_i^{L21}$ and $f_i^R$ are denoted by $g_i^L$ and $g_i^R$.

Following Mitchell and Shaefer (2000), they first define the expectation value of centroid of a fuzzy number $A_i$ as follows

$$M_i = \frac{\int_{c_i-l_i}^{c_i+r_i} x f_{A_i}(x) dx}{\int_{c_i-l_i}^{c_i+r_i} f_{A_i}(x) dx}$$ (22)

Furthermore, the transfer coefficient of $A_i$, $i = 1, \ldots, n$, is computed as in Wang et al. (2009) by

$$\lambda_i = \frac{M_i - M_{\text{min}}}{M_{\text{max}} - M_{\text{min}}}$$ (23)

where $M_{\text{max}} = \max\{M_1, \ldots, M_n\}$ and $M_{\text{min}} = \min\{M_1, \ldots, M_n\}$

$L$ and $R$ state for left and right respectively
Finally, they define the areas $s^L_i$ and $s^R_i$ of the left and the right sides of the fuzzy number $A_i$, $i = 1, ..., n$, as follows

$$s^L_i = \int_0^1 (g^L_{A_i}(y) - a_{min}) dy$$
$$s^R_i = \int_0^1 (d_{max} - g^R_{A_i}(y)) dy$$

(24) (25)

where $a_{min} = \min\{c_1 - l_1, ..., c_n - l_n\}$ and $d_{max} = \max\{c_1 + r_1, ..., c_n + r_n\}$.

Based on $\lambda_i$, $s^L_i$ and $s^R_i$, the ranking index of $A_i$ is defined as follows

$$s_i = \frac{s^L_i \lambda_i}{1 - s^R_i (1 - \lambda_i)}$$

(26)

Finally, the ranking is given, under certain condition by

**Definition 5.1 (Nejat and Mashinchi (2011))** For any two numbers, $A_i$ and $A_j$, based on (26), their order is defined by:

1. $A_i \succ A_j$, if and only if $s_i > s_j$
2. $A_i \prec A_j$, if and only if $s_i < s_j$
3. $A_i \sim A_j$, if and only if $s_i = s_j$

### 6 Empirical studies

In order to illustrate the fuzzy adjusted performance measures introduced in Section 4, we carry out an empirical study on French hedge funds. The dataset is composed of daily prices of 50 hedge funds and the sample period covers January 2000 through December 2005. They are listed in Table 1 with their tickers used in the paper. The MSCI France index (MSCIF) is chosen as benchmark. Its monthly returns based on closing prices are statistically characterized by 0.0036, 0.0571, 0.1072 and 2.967 as mean value, standard deviation, skewness and kurtosis respectively.

Table 2 provides summary statistics for the selected hedge funds monthly returns based on closing prices. We compute the mean, the standard deviation, the skewness and the kurtosis. The obtained values validate that the fund’s returns generally have left skewed distributions with negative mean value. These values emphasize the non-normal distribution of the returns. Using these returns, we compute the classical systematic risk (beta), the Jensen’s alpha, the Sharpe ratio, the Treynor ratio and the information ratio of the fifty funds. The considered funds are all defensive (beta less than 1) relatively to the benchmark. The performance measures displayed in the Table 8 are negative except

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22We use the following formulae for the computation of the classical ratios:

- systematic risk $\beta_i = \frac{Cov[R_i, R_m]}{\sqrt{Var[R_m]}}$
- Sharpe ratio $SR_i = \frac{E[R_i] - r_f}{\sqrt{Var[R_i]}}$
- Treynor ratio $TR_i = \frac{E[R_i] - r_f}{\delta_i}$
- Jensen’s alpha $\alpha_i = E[R_i] - \beta_i E[R_m]$
- information ratio $IR_i = \frac{E[R_i - R_m]}{\sqrt{Var[R_i - R_m]}}$
Table 1: This table lists the hedge funds and their tickers used in the empirical studies.

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for 18% of funds relatively for the Sharpe ratio, 18% of funds for the Treynor ratio, 14% of funds for the information ratio and 12% of funds for the Jensen’s alpha. Only AvipTop, BarcWEP, ChauInt, ElMult and GestPriv exhibit all positive adjusted performance measures.

Following the procedure described in Section 3, we compute the triangular fuzzy returns. These fuzzy returns are then used to estimate the fuzzy versions of systematic risk, Jensen’s alpha, Sharpe ratio, Treynor ratio and information ratio. As stated throughout Section 4, the fuzzy version of the adjusted performance measures have the same shape functions as the returns; i.e. symmetric triangular membership functions. Table 7 displays the center and the spreads of the all fuzzy adjusted performance measures. In the remainder of the section, we derive and compare the rankings of hedge funds according to the crisp performance ratios and the fuzzy risk adjusted measures.

Following the ranking method for fuzzy numbers presented in Section 5, we order the funds relatively to the fuzzy Sharpe ratio. Table 7 provides this ranking and also the ranking relative to the classical crisp Sharpe ratio and the membership degree of this latter to fuzzy Sharpe ratio. These membership degrees are close to 1 (more than 0.9183) hence the centered position of the crisp Sharpe ratios on the supports of the fuzzy ones. We also observe that 24% of funds undergo modification of their rank and the change in their ranking is of order 1 (expect 2 for Fund N° 31), that is, a hedge fund does not lose or gain more than 1 rank points. In addition, we remark that these changes are observed in the first half on the ranking. One can conclude that even if the fuzzy approach produces a change in ranking, this change is not material.

Table 4 provides the funds ranking based on the crisp and fuzzy Treynor ratios. As in the case
Table 2: This table gives descriptive statistics of monthly returns based on closing prices of all used hedge funds.

Table 5 provides the ranking according to Jensen’s alpha and its fuzzy version. The membership degrees of the crisp Jensen’s alpha to the fuzzy ones are very close to 1 (more than 0.9999) expect for the Fund N° 16, which presents a null membership degree. The two rankings are very similar and only Funds N° 14 swaps position with Fund N° 49 by losing and gaining one rank point, respectively, with the fuzzy approach.

Table 6 provides the funds ranking based on the crisp and fuzzy information ratios. Contrarily to the Sharpe and Treynor ratio case, there exist some null membership degrees for the crisp ratio to their fuzzy set-valued versions, for 6 funds (N° 34, 43, 25, 16, 45 and 23). Other crisp ratios have membership degrees superior to 0.8003. The resulting rankings show that 18% of funds see their rank change and the difference of rankings fluctuate between 1 and 3. Funds N° 25, 46 and 47 gain one rank point while Funds N°41 and 50, gain 2 ranks. At the same time, Funds N° 6 and 16 loose one rank point whereas Funds N° 29 and 35 decrease their positions for 2 and 3 points respectively.

i.e., the crisp Jensen’s alpha does not belong to the support of its fuzzy version.
To summarize our discussion, the fuzzy approach first allows us associate closing prices-based returns to a summary statistic relating to the market friction noise that it contains. This fuzzy representation aims at reflecting the imprecise nature of the hedge funds observed returns. The fuzzy returns are then used for the fuzzy formulation of the Sharpe ratio, Treynor ratio, Jensen’s alpha and information ratio. The spreads of the fuzzy ratios, induced by the imprecision of the returns, is penalized in the funds ranking. This ranking exhibits some difference with the classical approach, specially, for the Sharpe, Treynor and information ratios.

7 Conclusions
Hedge funds performance evaluation requires an accurate determination of the sources of uncertainty. The common probabilistic approach derives from the CAPM and assumes that the returns are random variables normally distributed and linearly linked to the risk factors. However, the vagueness and the bias on the published results of hedge funds, as well as the violation of the normal distribution assumption highlighted by some empirical studies, suggest the use of a fuzzy formulation of performance ratios. This paper proposes a fuzzy formulation of Sharpe ratio, Treynor ratio, Jensen’s alpha and the information ratio.

The fuzzification of the returns is however the first step in defining the fuzzy adjusted performance measures. Fuzziness aims to express the effect of noise induced by market microstructure frictions on the observed return. For that purpose, the spread of a fuzzy return is estimated as the standard deviation of the noise and its central value is represented by the observed return. This is a convenient way to define the fuzzy return in the sense that it combines a statistic summary of the noise and the commonly use return value.

The fuzzy ratios measure the performance of the hedge funds by taking into account the fuzziness of the returns and their stochastic variability. These fuzzy set-valued adjusted performance measures have the advantage of combining the two most known sources of uncertainty. For our data, the fuzzy ratios boundaries generally contain the crisp ratios and the resulting ranking show changes funds position specially for Sharpe, Treynor and Information ratios.
References


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Table 3: This table compares the Sharpe ratio ranking with the fuzzy and with the classical approaches.
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Table 6: This table compares the information ratio ranking with the fuzzy and with the classical approaches.
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Table 7: This table reports the triangular fuzzy performance ratio of the second half of the studied funds.
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Table 8: This table reports the systematic risks and the adjusted performance measures based on monthly returns of hedge funds indices.
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Stéphane MUSSARD : mussard@lameta.univ-montp1.fr