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Measurements and properties of the values of time and reliability

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Abstract:

This paper derives new monetary measures of traveler's willingness to pay to save travel time and to improve its reliability. We develop an intuitive model of transport mode choice in which each alternative is fully characterized by its price and the distribution of its random travel time, assuming expected utility preferences over the latter. Hence, the value of time (VOT) and the value of reliability (VOR) are defined and theirs properties are established. Finally, we use data from a discrete choice experiment in stated preferences to illustrate how our measures can provide behavioral estimations of the VOT and the VOR.

Key words: Value of travel time savings, value of reliability, risk attitudes, prudence, cost-benefit analysis of transport infrastructure projects.

1.0 Introduction

The improvement of transport systems is an important public policy issue for virtually any government. In practice, public decisions regarding transport policy usually rely on the cost-benefit analysis of alternative projects. Among the benefits of an improved transport system, it is now well-established that the value of travel time (VOT) and the value of travel time reliability (VOR) for users are two very important elements. In general, the appropriate appraisal of almost any transport project requires theoretical measures and empirical valuations of both the VOT and the VOR. This is the concern of the present paper.

The value of time has a long history in microeconomic theory, dating back at least to the seminal contribution on the optimal allocation of time by Becker (1965), where time appears as an input for the consumption of final goods (from which utility is ultimately derived). In Becker's framework, the value of time is simply the wage rate, that is the cost of not earning money during an out-of-work time period. Shortly after, Johnson (1966) explicitly incorporated work time into preferences, that is as an argument of the consumer's utility function, and concluded that the VOT was equal to the sum of two terms: the wage rate and a monetary value of the marginal utility of work time. Going a step further, Oort (1969) suggested that travel time was undesirable in itself, and should therefore also be incorporated into preferences. Building on these works, by adding scheduling considerations to both the utility function and the constraints, Small (1982) introduced the now standard scheduling model.

On the other hand, by relaxing the assumption of a sure or fully reliable travel time, with the aim to provide theoretical foundations to the so-called safety margin, Gaver (1968) and Knight (1974) have been the first to incorporate travel time reliability considerations in departure time choice models. Subsequently, assuming mean-variance preferences over random travel time, Jackson and Jucker (1981) were able to explain data collected from a discrete choice experiment in stated preferences in which subjects were asked to choose among risky travel alternatives. It is worth-mentioning that all these early contributions only implicitly rely on the assumption of von Neumann and Morgenstern (1947)'s expected utility preferences over random travel time. In this respect, notable advances are due to Polak (1987) and Senna (1994), who derive measures of both the VOT and the VOR in the context of a general model of travel choice, and to Noland and Small (1995) and Noland and al. (1998), who extended Small (1982)'s scheduling model to accommodate reliability considerations through the expected utility assumption. Without being exhaustive, more recent contributions include Bates and al. (2001), Blayac and Causse (2001), Lam and Small (2001), Brownstone and Small (2005), Small and al. (2005), Batley (2007), Asensio and Matas (2008), Jong and al. (2009), Fosgerau and Karlström (2010), Fosgerau and Engelson (2011), Hensher and al. (2011) and Devarasetty and al. (2013).[1]

In the present paper, we follow the transportation literature by accommodating travel time reliability through the expected utility assumption. We develop a simple general model of mode choice in which each alternative is fully characterized by its price and the distribution of its travel time. The preferences function of travelers is assumed separable and quasi-linear. It is the sum of a linear function of the price and of a non-linear expected utility function of the variable travel time. In this context, a traveler is said to be reliability-prone if he/she prefers a fully reliable alternative to a risky one whenever both have the same price and the same mean travel time. As one would have expected, reliability proneness is equivalent to the concavity of the preferences function with respect to travel time, in other words, the marginal utility of travel time is decreasing. If the marginal utility of travel time is also concave, then travelers are said to be prudent. We then introduce model-free definitions of the VOT and of the VOR. The VOT is defined as the willingness to pay for a given reduction in travel time and the

[1]Useful reviews of the literature can be found in Wardman (1998), Noland and Polak (2002), Jong and al. (2004), Li and al. (2010) and Carrion and Levinson (2012).

VOR is defined as the willingness to pay for a full elimination of the variability of the travel time. This contrast with most of the transportation literature, where the VOT and the VOR are typically defined as marginal rates of substitution, between the mean travel time and wealth for the VOT, and between a variable representing the variability of the travel time (typically its variance) and wealth for the VOR. As a result, almost all existing measures of the VOT and the VOR only give travelers' willingness to pay for small travel time savings and for small improvements in the reliability of travel time.[2] Furthermore, in many studies, this problem is eliminated by assuming that travelers' preferences function is linear over travel time.

Because we consider a non-linear expected utility preferences function over travel time, our measure of the VOT is not necessarily constant with travel time. Therefore, the model allows travelers to be ready to pay more or less for reducing the travel time of more longer trips. In addition, our measure of the VOT depends on the variability of travel time. In particular, we show that, for all reliability-prone travelers, the VOT is increased by any first-order stochastic deterioration in the distribution of travel time. For example, this result implies that all reliability-prone travelers should be ready to pay less to save ten minutes on a one-hour trip rather than on a two-hours trip. In addition, we show that if travelers are also prudent, then their VOT is increased by any second-order stochastic deterioration in the distribution of travel time. For example, this implies that all reliability-prone and prudent travelers should be ready to pay less to save ten minutes on a one-hour trip, rather than on a risky trip with a mean travel time of one hour. A key intuition to understand these results is that reliability proneness and prudence characterize a general preference for the combination of good things (such as travel time savings) with bad things (such as more travel time or more variability of

[2] A notable exception is Batley (2007) who has introduced the concept of reliability premium, by reference to Pratt (1964)'s risk premium, in a scheduling model. However, Batley (2007)'s reliability premium is expressed in time units.

travel time).[3] Thus, travelers with such preferences would be ready to pay more to save travel time in transport with an already high and/or unreliable travel time. We also propose two complementary measures of travelers' willingness to pay to escape all the variability of the travel time, one expressed in time units, called the reliability premium, and one expressed in monetary units, that is the VOR. Locally, we show that the reliability premium is equal to the so called reliability ratio defined as the ratio of the VOR and the VOT. Of course the VOR is positive for all reliability-prone travelers and it is increasing with both their degree of reliability proneness and the level of actuarially neutral risk affecting travel time. We also show that keeping the level of risk constant, the VOR is increasing with the mean travel time if travelers exhibit increasing absolute reliability proneness.

Finally, we show how our theoretical measures can be easily implemented to provide empirical valuations of the VOT and of the VOR. We illustrate this by using data from a discrete choice experiment in stated preferences framed for personal long distance trips through high-speed rail transportation in France (TGV). The estimation results from a conditional logit regression yield a VOT between 11 € and 57 € for one hour of travel time saved and a VOR between 12 € and 16 € for one hour of standard deviation of travel time saved.

The rest of the paper is organized as follows. Section 2 presents the theoretical framework and introduces the concepts of reliability proneness and of prudence. In section 3, we define the VOT and we establish the link between reliability proneness and the shape of the VOT. In section 4, we present and explore the properties of our two complementary measures for the valuation of travel time reliability (the reliability premium and the VOR). In section 5, we discuss the implication of the choice of a particular functional form for the utility function.

[3] For more on this interpretation see Eeckhoudt and Schlesinger (2006), Eeckhoudt and al. (2009), Crainich and al. (2011).

We briefly present our data and the results of the econometric estimation of the model. Section 6 concludes.

2.0 Traveler's attitudes towards travel time variability

2.1 Basic model and assumptions

Consider a traveler, or any user of a transport system, who wishes to reach a destination. Suppose that he/she has the possibility to make a choice among a given number of alternatives. Each alternative i is fully characterized by its cost $c_i > 0$ and its random travel time $\tilde{t}_i$. For convenience, we assume that all the random variable we consider have the bounded interval $\left[t^{min}, t^{max}\right] \subset \mathbb{R}_+$ as support.[4] Letting $F_i(t) = \Pr[\tilde{t}_i \leq t]$ be the CDF of $\tilde{t}_i$, the mean travel time is

$$\mu_i = E\tilde{t}_i = \int_{t^{min}}^{t^{max}} td\, F_i(t) \tag{1}$$

And the variance of travel time is

$$\sigma_i^2 = E\tilde{t}_i^2 - \mu_i^2 = \int_{t^{min}}^{t^{max}} [t^2 - \mu_i^2] d\, F_i(t) \tag{2}$$

We now assume that the traveler's preferences over alternatives can be represented by a quasi-linear preferences function of the expected utility form:

$$U_i = EU(c_i, \tilde{t}_i) = -\lambda c_i + Eu(\tilde{t}_i) \tag{3}$$

where $\lambda > 0$ represents the constant marginal utility of wealth and $u: \mathbb{R}_+ \to \mathbb{R}$ is a differentiable non-linear utility function of travel time which plays exactly the same role than the Bernoulli (1738)'s utility function in decision theory under monetary risk.[5] The expected utility of travel time is given by

$$u_i = Eu(\tilde{t}_i) = \int_{t^{min}}^{t^{max}} u(t) d\, F_i(t) \tag{4}$$

[4] Since any random variable $\tilde{t}_i$ may be expressed as $t_i + \tilde{\epsilon}_i$, where t_i is a constant and $\tilde{\epsilon}_i$ is a random variable distributed as $\tilde{t}_i$, two interpretations are possible: either t_i is the mean travel time and $\tilde{\epsilon}_i$ is a zero-mean risk, or t_i is the minimum possible travel time and $\tilde{\epsilon}_i$ is a non-negative risk.

[5] Because the preferences function is supposed quasi-linear, the marginal utility of travel time is independent of the cost of the alternative.

As is well-known, the sign of the first derivative of the utility function establishes a relationship with a stochastic order named the first-degree stochastic dominance (FSD) order. Consider two transport alternatives with the same cost, say alternative i with travel time $\tilde{t}_i$and alternative j with travel time $\tilde{t}_j$. By definition, $\tilde{t}_j$is a FSD deterioration of $\tilde{t}_i$ if and only if the probability to experience a smaller travel time than any given one is always greater with the alternative i than with the alternative j, that is$F_i \geq F_j$ everywhere.

On the one hand, we could search under which condition all travelers who dislike more travel time would prefer the alternative i to the alternative j. On the other hand, we could search under which condition all travelers would prefer the alternative i to the alternative j whenever $\tilde{t}_j$is a FSD deterioration of $\tilde{t}_i$. Fortunately, proposition 1 states that these two approaches are in fact equivalent.

Proposition 1 *The following statements are equivalent:*

(i) *All travelers who dislike more travel time prefer $\tilde{t}_i$ to $\tilde{t}_j$.*

(ii) *$\tilde{t}_j$is a FSD deterioration of$\tilde{t}_i$.*

A direct implication of proposition 1 is that FSD deteriorations in travel time should be disliked by a wide set of travelers. It is obvious that all travelers who dislike more travel time also dislike FSD deteriorations. Indeed, FSD deteriorations involve a transfer of probability mass to the right, that is from low travel time states of the world to high travel time states of the world, or, equivalently, by adding some non-negative risk to travel time. Thus any FSD deterioration implies a greater mean travel time. For instance, all travelers who dislike more travel time should prefer a sure travel time of 120 min to $\tilde{t}_1$ =(120 min, ½; 150 min, ½),or similarly they dislike the non-negative risk$\tilde{\varepsilon}_1$ =(0 min, ½; 30 min, ½). More generally, proposition 1 tells us that all travelers who dislike more travel time should also prefer $\tilde{t}_1$to $\tilde{t}_2$= (120 min, ¼; 150 min, ¾) and $\tilde{t}_1$to $\tilde{t}_3$= (120 min, ½; 150 min, ¼; 180 min, ¼) where $\tilde{t}_2$and $\tilde{t}_3$are obtained from $\tilde{t}_1$by adding the non-negative risk$\tilde{\varepsilon}_1$ either to the low or to the high travel time realization of $\tilde{t}_1$, respectively. Of course, the FSD order is incomplete,[6] but when it works, it provides a normatively appealing criterion to rank any two transport alternatives having the same cost.

2.2 Reliability proneness

To deal with travelers' attitudes towards travel time variability, we begin with a model-free definition of reliability proneness which mimic the standard definition of risk aversion.

Definition 2 *A traveler is reliability-prone whenever he/she always prefers an alternative with a fully reliable travel time to an alternative with a risky travel time whenever both alternatives feature the same cost and the same mean travel time.*

Under the expected utility assumption, Arrow (1965) and Pratt (1964) have demonstrated that risk aversion is equivalent to the concavity of the Bernoulli (1738)'s utility function. Proposition 3 below tells us that this result applies for reliability proneness as well.

Proposition 3 *The following statements are equivalent:*

(iii) *The traveler is reliability-prone*

(iv) *The traveler's preferences function is a concave function of travel time.*

Observe that definition 2 is equivalent to say that reliability-prone travelers dislike any actuarially-neutral risk affecting the distribution of travel time. In other words, reliability-prone travelers dislike any increase in risk in the sense of Rothschild and Stiglitz (1970). By definition, $\tilde{t}_j$ contains more actuarially-neutral risk than $\tilde{t}_i$ if and only if:[7]

$$\int_t^{t^{max}} F_i(s)ds \geq \int_t^{t^{max}} F_j(s)ds \text{ for all } t \text{ and } \mu_i = \mu_j \qquad (5)$$

Proposition 4 *The following statements are equivalent:*

(v) *All reliability-prone travelers prefer $\tilde{t}_i$to $\tilde{t}_j$.*

[6] In particular, $\tilde{t}_2$ and $\tilde{t}_3$ cannot be ranked according to the FSD criterion.

[7] Because travel time is a non-desirable good, we integrate from t to t^{max}, rather than from t^{min} to t as in the standard definition of an increase in risk.

(vi) $\tilde{t}_j$ *contains more actuarially-neutral risk than* $\tilde{t}_i$.

For instance, proposition 4 tells us that all reliability-prone travelers should prefer a sure travel time of 135 min to a random travel time $\tilde{t}_1$ =(120 min, ½; 150 min, ½), that is they dislike the actuarially-neutral risk $\tilde{\epsilon}_2$ =(-15 min, ½; 15 min, ½). In addition, all reliability-prone travelers should prefer $\tilde{t}_1$ to $\tilde{t}_4$= (105 min, ¼; 135 min, ¼; 150 min, ½) and $\tilde{t}_1$ to $\tilde{t}_5$ = (120 min, ½; 135 min, ¼; 165 min, ¼), where $\tilde{t}_4$ and $\tilde{t}_5$ are obtained from $\tilde{t}_1$ by adding the actuarially-neutral risk $\tilde{\epsilon}_2$ either to the low or to the high travel time realization of $\tilde{t}_1$, respectively. By contrast with FSD deteriorations, it is less obvious that all real-world travelers dislike any actuarially-neutral increase in risk. This is so because the travel time could be smaller in some states of the world.

More generally, taking as given that they dislike more travel time, we show that all reliability-prone travelers respect the second-degree stochastic dominance (SSD) order. Indeed, a SSD deterioration can be obtained by combining any FSD deterioration with any actuarially-neutral increase in risk. It is worth- mentioning that a SSD deterioration implies an increase in the variance of travel time but the reverse is not necessarily true. Thus, proposition 5 below proves that there exists a link between the SSD order and the sign of the first two successive derivatives of travelers' preferences function with respect to travel time.

Proposition 5 *The following statements are equivalent:*

(vii) *All reliability-prone travelers who dislike more travel time prefer* $\tilde{t}_i$ *to* $\tilde{t}_j$.

(viii) $\tilde{t}_j$ *is a SSD deterioration of* $\tilde{t}_i$.

Finally, observe that Arrow (1965) and Pratt (1964) have also demonstrated that the degree of concavity of the Bernoulli (1738)'s utility function – captured by the absolute risk aversion function defined as minus the ratio of its second-derivative and its first-derivative – governs the behavior of decision-makers facing monetary risks. In the present context, for a traveler with preferences over travel time characterized by a strictly decreasing utility function u, we define the absolute reliability proneness function as

$$r(t) = \frac{u''(t)}{u'(t)} \tag{6}$$

Taking as given that travelers dislike more travel time, reliability proneness is equivalent to $r \geq 0$ everywhere. Proposition 7 is simply obtained by adapting Pratt (1964)'s fundamental theorem on comparative risk aversion. In the present context, proposition 7 states that the absolute reliability proneness function r provides an indicator of the intensity of travelers' aversion towards travel time variability.

Definition 6 *Consider two travelers with decreasing utility functions u and v, say traveler u and traveler v. Traveler v is more reliability-prone than traveler u if traveler v dislikes any risk affecting travel time that traveler u dislikes.*

Proposition 7 *The following statements are equivalent:*

(ix) *Traveler v is more reliability-prone than traveler u.*

(x) *r is uniformly greater for traveler v than for traveler u.*

(xi) *v is a concave transformation of u.*

It is important to note that comparative reliability proneness can be applied to a single traveler. Indeed, while all reliability-prone travelers dislike travel time variability, they may find a given risk more or less harmful depending on whether it affects an alternative with a longer travel time. In other words, reliability-proneness may be increasing, constant or decreasing with travel time. Thus, at least three kinds of preferences can be distinguished. Travelers exhibiting increasing absolute reliability proneness (IARP) should find any risk more harmful when it affects an alternative with a longer travel time. On the other hand, travelers exhibiting decreasing absolute reliability proneness (DARP) should find any risk less harmful when it affects an alternative with a longer travel time. Finally, travelers exhibiting constant absolute reliability proneness (CARP) should find any risk as equally harmful independently of the travel time length. As shown in section 5 below, the functional forms that are commonly used in the transportation literature, such as quadratic and power utility functions, all impose DARP.

2.3 Prudence

Going one step further, let us consider the implications of the sign of the third derivative of the preferences function with respect to travel time. As shown hereafter, this sign is important because it determines the shape of both the VOT and the VOR.

Definition 8 *A traveler is prudent if his/her marginal utility of travel time is concave.*

In the expected utility literature, a positive sign of the third derivative of the Bernoulli (1738)'s utility function defines what is called 'prudence' and characterizes an aversion to any increase in downside risk.[8] By contrast, because travel time is here a non-desirable good, prudence is here consistent with a negative third-derivative of the preferences function with respect to travel time and characterizes an aversion to any increase in upside risk. Formally, $\tilde{t}_j$contains more upside risk than $\tilde{t}_i$ if and only if

$$\int_t^{t^{max}} \int_s^{t^{max}} \left[F_i(\ell) - F_j(\ell)\right] d\ell ds \geq 0 \text{ for all } t \text{ and } \mu_i = \mu_j \text{ and } \sigma_i^2 = \sigma_j^2 \qquad (7)$$

Proposition 9 *The following statements are equivalent:*

(xii) *All prudent travelers prefer $\tilde{t}_i$to $\tilde{t}_j$.*

(xiii) *$\tilde{t}_j$ contains more upside risk than $\tilde{t}_i$.*

By definition, changes in upside risk preserve both the mean and the variance of the random variable. To illustrate the implications of prudence, consider again time $\tilde{t}_1 =$ (120 min, ½; 150 min, ½) and suppose that the actuarially-neutral risk $\tilde{\epsilon}_2 =$ (-15 min, ½; 15 min, ½) can be added either to the low (120 min) or to the high (150 min) travel time realization of $\tilde{t}_1$. A prudent traveler should prefer the former option. Equivalently, he or she should prefer $\tilde{t}_4$

[8] The notion of prudence was introduced by Kimball (1990) to measure the intensity of saving in the face of a future risk on wealth.

=(105 min, ¼; 135 min, ¼; 150 min, ½), where $\tilde{\epsilon}_2$has been added to the low travel time realization of $\tilde{t}_1$ to $\tilde{t}_5$ = (120 min, ½; 135 min, ¼; 165 min, ¼), where $\tilde{\epsilon}_3$ has been added to the high travel time realization of $\tilde{t}_1$. Observe that $\tilde{t}_4$ and $\tilde{t}_5$ have the same mean (μ_4 = μ_5 = 135 min) and the same variance (σ_4^2 = σ_5^2 = 337.5 min), but $\tilde{t}_4$ contains more upside risk than $\tilde{t}_5$. Intuitively, reliability-prone and prudent travelers are willing to combine good things (such as a low travel time) with bad things (such as a risk on travel time).[9] By contrast, reliability-prone travelers exhibiting DARP should find $\tilde{\epsilon}_2$ more harmful if it affects a sure travel time of 120 min rather than a sure travel time of 150 min. This may help to recognize that DARP works against prudence. Indeed, the absolute prudence function is defined as

$$p(t) = \frac{u'''(t)}{u''(t)} \tag{8}$$

Then observe that the sign of r' is equal to the sign of $p - r$. Thus, $p \leq r$ everywhere is equivalent to DARP, $p \geq r$ everywhere is equivalent to IARP and $p = r$ everywhere is equivalent to CARP. On the one hand, all travelers exhibiting IARP are prudent, but the reverse is not true. Hence, prudent travelers may either exhibit DARP, IARP or CARP. Moreover, all non-prudent travelers exhibit DARP. Thus, under DARP, travelers are allowed to be prudent, but not too much.

Finally, using the same reasoning than the one we used to establish proposition 5, we get that all reliability-prone and prudent travelers who dislike more travel time respect the third-degree stochastic dominance (TSD) order. Indeed, a TSD deterioration can be obtained by combining any SSD deterioration with any increase in upside risk. Thus an increase in upside risk is a particular TSD deterioration, as an actuarially-neutral increase in risk is a particular SSD.

[9] Reliability-averse travelers may also consistently be prudent. Indeed, because an actuarially-neutral risk is a good thing for reliability-averse travelers and because such travelers prefer to combine good things with good things, they would also prefer to allocate the actuarially-neutral risk $\tilde{\epsilon}_2$ (a good thing) to the low travel time realization of $\tilde{t}_1$ (a good thing as well).

Proposition 10 *The following statements are equivalent:*

(xiv) *All reliability-prone and prudent travelers who dislike more travel time prefer* $\tilde{t}_i$ *to* $\tilde{t}_j$.

(xv) $\tilde{t}_j$ *is a TSD deterioration of* $\tilde{t}_i$.

3.0 Travel time variability and the value of travel time

When the travel time of an alternative is random, the VOT reflects travelers' willingness to pay to reduce the mean travel time of the alternative. There are, however, numerous ways in which this can be done. Here, the random travel time is supposed to be reduced by the same positive amount in all states of the world. Thus, the VOT is defined for a sure travel time saving arising, for instance, through a reduction in the free-flow travel time.

Definition 11 *The VOT is the maximum monetary amount that a traveler is ready to pay to save* $n > 0$ *unit(s) of total travel time for sure in all states of the world.*

According to definition 11, the VOT for an alternative i is implicitly defined by the following equality:

$$EU(c_i, \tilde{t}_i) = EU(c_i + VOT_i, \tilde{t}_i - n) \tag{9}$$

From (3) and (4), we get an explicit formula:

$$VOT_i = EVOT(\tilde{t}_i) = \int_{t^{min}}^{t^{max}} VOT(t) dF_i(t) \tag{10}$$

where

$$VOT(t) = \frac{u(t-n) - u(t)}{\lambda} \approx -n \frac{u'(t)}{\lambda} \tag{11}$$

gives the VOT for a given sure travel time t. Clearly the VOT is positive whenever the preferences function is decreasing with both the cost and the travel time of alternatives. Recalling that λ represents the marginal utility of wealth, the VOT is a monetary measure of the traveler's welfare gain from saving n unit(s) of travel time. Therefore, the VOT will be greater for travelers with a low marginal utility of wealth. As a result, if the marginal utility of wealth is decreasing, then the VOT would be greater for richer travelers, as the common sense suggests.

A more interesting point is whether the VOT is decreasing or increasing with travel time. Would a traveler be ready to pay more or less to save travel time when the mean travel time is greater? Although this is ultimately an empirical question, it is important to know under which conditions, regarding the form of travelers' preferences function, each property holds. The following result gives a clear answer to this question.

Proposition 12 *The following statements are equivalent:*

(xvi) *For all reliability-prone travelers, the VOT is greater for* $\tilde{t}_j$ *than for* $\tilde{t}_i$.

(ii) $\tilde{t}_j$ *is a FSD deterioration of* $\tilde{t}_i$.

In the particular case where an alternative features a sure travel time, a FSD deterioration is simply equivalent to an increase in travel time. Thus, proposition 12 implies that all reliability-prone travelers would be ready to pay more to reduce the travel time of more longer trips. Roughly speaking, reliability proneness implies that the VOT is increasing with travel time. This follows from the fact that reliability-prone travelers have a negative and decreasing marginal utility of travel time. By consequence, reliability-prone travelers value more travel time savings at higher levels of travel time.

Moreover, we can also determine under which condition the VOT is greater or smaller for a risky trip than for a sure trip whenever both have the same mean. Proposition 13 below tells us that the VOT of all prudent travelers is increasing with the variability of travel time.

Proposition 13 *The following statements are equivalent:*

(xvii) *For all prudent travelers, the VOT is greater for* $\tilde{t}_j$ *than for* $\tilde{t}_i$.

(vi) $\tilde{t}_j$ *contains more actuarially-neutral risk than* $\tilde{t}_i$.

According to proposition 13, all prudent travelers would be ready to pay more to reduce the travel time of more risky trips. Thus, their VOT is increasing with the variability of travel time. For example, prudent travelers would be ready to pay more to reduce by 15 min a random travel time $\tilde{t}_1$= (120 min, ½; 150 min, ½) than to reduce by 15 min a sure travel time of 135 min. They would thus prefer a change from $\tilde{t}_1$ to $\tilde{t}_6$ = (105 min, ½; 135 min, ½), where $\tilde{t}_6$ is obtained from $\tilde{t}_1$ by substracting 15 min to each possible realization of $\tilde{t}_1$, to a change from 135 min to 120 min. To better understand why this property of the VOT is linked to the sign of the third-derivative of the utility function, it may be observed from (10) that the VOT is calculated as an expectation. In particular, the VOT is greater for the random travel time $\tilde{t}_1$ than for a sure travel time of 135 min if and only if

$$VOT_1 = \frac{1}{2}VOT(120) + \frac{1}{2}VOT(150) \geq VOT(135) \tag{12}$$

By Jensen (1906)'s inequality, we know that this is true if and only if the VOT is convex, which is equivalent to prudence.

To illustrate further, consider any random travel time $\tilde{t}_i$ expressed as $\mu_i + \tilde{\epsilon}$, where μ_i is the mean travel time and $\tilde{\epsilon}$ is an actuarially-neutral risk. Now suppose, that $\tilde{\epsilon}$ is sufficiently small such that its variance σ_i^2 tends towards zero but less rapidly than higher moments (skewness, kurtosis, etc.). Expanding the VOT around the mean travel time we get an instructive approximation:[10]

$$VOT_i \approx VOT(\mu_i) - \frac{1}{2}\sigma_i^2 \frac{u'''(\mu_i)}{\lambda} \tag{13}$$

This formula makes explicit the separate effects on the VOT of the mean travel time, captured by the first term, and of the variability of travel time, captured by the second term. The first term is simply the VOT calculated for a sure travel time equal to the mean travel time of the alternative. The second term is (minus) half the product of the variance of travel time and of the third-derivative of the utility function (evaluated at the mean travel time of the alternative)

[10] If n is equal to one small unit of time, we have $VOT(t) \approx -u'(t)/\lambda$. Expanding u' around μ_i, we get $u'(\tilde{t}_i) = u'(\mu_i + \tilde{\epsilon}) \approx u'(\mu_i) + \tilde{\epsilon}u''(\mu_i) + \frac{1}{2}\tilde{\epsilon}^2 u'''(\mu_i)$. Thus $VOT_i \approx Eu'(\tilde{t}_i)/\lambda \approx u'(\mu_i)/\lambda + \frac{1}{2}\sigma_i^2 u'''(\mu_i)/\lambda$.

divided by the marginal utility of wealth. Again, it is clear that the variability of travel time always raises the VOT if and only if the traveler is prudent with $u''' \leq 0$ everywhere.

Finally, recalling that any SSD deterioration in the distribution of travel time can be obtained by combining any FSD deterioration with any actuarially-neutral increase in risk, proposition 14 below directly follows from propositions 12 and 13.

Proposition 14 *The following statements are equivalent:*

(xviii) *For all reliability-prone and prudent travelers, the VOT is greater for $\tilde{t}_j$ than for $\tilde{t}_i$.*

(viii) *$\tilde{t}_j$ is a SSD deterioration of $\tilde{t}_i$.*

4.0 Two measures of the value of travel time reliability

4.1 The reliability premium

We first consider a non-monetary measure of the value of travel time reliability which is equivalent to the one proposed by Batley (2007). Because it is similar to the Arrow-Pratt's risk premium, it is called the reliability premium. Observe that, by contrast with the VOR defined hereafter, the reliability premium is expressed in time units and is therefore a non-monetary measure of the value of travel time reliability.

Definition 15 *The reliability premium Π_i is the maximum amount of additional travel time that a traveler is ready to accept to escape all the variability of the travel time of an alternative i, that is to get a sure travel time equal to the mean travel time of the alternative plus the reliability premium.*

According to definition 15, the reliability premium Π_i for an alternative i is implicitly defined by the following equality:

$$EU(c_i, \tilde{t}_i) = U(c_i, \mu_i + \Pi_i) \tag{14}$$

Because the reliability premium is expressed in time units, it is not possible to obtain an explicit formula. Using (3) and (4), the above condition is equivalent to

$$Eu(\tilde{t}_i) = u(\mu_i + \Pi_i) \tag{15}$$

Clearly, the reliability premium is positive for all reliability-prone travelers who have a concave utility function. In addition, the reliability premium increasing with any actuarially-neutral increase in risk.

Proposition 16 *The following statements are equivalent:*

(xix) *For all reliability-prone travelers who dislike more travel time, the reliability premium is greater for $\tilde{t}_j$ than for $\tilde{t}_i$.*

(vi) *$\tilde{t}_j$contains more actuarially-neutral risk than $\tilde{t}_i$.*

As above, consider any alternative i with random travel time $\tilde{t}_i$ expressed as $\mu_i + \tilde{\epsilon}$, where μ_i is the mean travel time and $\tilde{\epsilon}$ is a small actuarially-neutral risk. We thus get an approximation of the reliability premium which is identical to Pratt (1964)'s famous approximation of the risk premium:[11]

$$\Pi_i \approx \frac{1}{2}\sigma_i^2 r\left(\mu_i\right) \qquad (16)$$

The reliability premium is approximately half the product of the variance of travel time and of the absolute reliability proneness function (evaluated at the mean travel time of the alternative). Taking as given that travelers dislike more travel time, reliability proneness is equivalent to $r \geq 0$ everywhere, insuring that the reliability premium is increasing with the variability of travel time. This illustrates proposition 16. In addition, the approximation of the reliability premium in (16) reveals that the value of the absolute reliability proneness function determine the value of the reliability premium. The more a traveler is reliability prone, the greater is his/her reliability premium. Using Pratt (1964)'s terminology, this is true in the small and in the large. Proposition 17 thus gives an additional statement to proposition 7.

Proposition 17 *The following statements are equivalent:*

(ix) *Traveler v is more reliability-prone than traveler u.*

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[11] Expanding u around μ, we get: $Eu(\tilde{t}_i) = Eu(\mu_i + \tilde{\epsilon}) \approx u(\mu_i) + \frac{1}{2}\sigma_i^2 u''(\mu_i)$ and $u(\mu_i + \Pi_i) \approx u(\mu_i) + \Pi_i u'(\mu_i)$.

(xx) *Traveler v's reliability premium is always greater than the one of traveler u.*

A direct consequence of proposition 17 is that the reliability premium is decreasing with travel time for all travelers exhibiting DARP. For instance, all reliability-prone travelers exhibiting DARP should find any risk such as less harmful as they affect a longer travel time.

4.2 The monetary value of travel time reliability

An important shortcoming of the reliability premium is that it is expressed in time units. Indeed, in practice, the cost-benefit analysis of transport infrastructure projects requires a monetary value of travel time variability, that is the VOR.

Definition 18 *The VOR is the maximum monetary amount that a traveler is ready to pay to escape all the variability of the travel time of an alternative, that is to get a sure travel time equal to the mean travel time of the alternative.*

According to definition 18, the VOR for a given alternative i is implicitly defined by the following equality:

$$EU(c_i, \tilde{t}_i) = U(c_i + VOR_i, \mu_i) \tag{17}$$

From (3) and (4), we get an explicit formula:

$$VOR_i = EVOR(\tilde{t}_i) = \int_{t^{min}}^{t^{max}} VOR(t)dF_i(t) \tag{18}$$

where

$$VOR(\tilde{t}_i) = \frac{u(\mu_i) - u(\tilde{t}_i)}{\lambda} \tag{19}$$

Thus, according to Jensen (1906)'s inequality, it is clear that the VOR is positive for all reliability-prone travelers whose preferences function is concave with travel time. More generally, we have the following result.

Proposition 19 *The following statements are equivalent:*

(xxi) *For all reliability-prone travelers, the VOR is greater for $\tilde{t}_j$ than for $\tilde{t}_i$.*

(vi) *$\tilde{t}_j$ contains more actuarially-neutral risk than $\tilde{t}_i$.*

Proposition 19 simply tells us that the VOR is increasing with the variability of travel time. However, the VOR is not necessarily increased by any SSD deterioration in the distribution of travel time. The reason is that SSD deteriorations (which include FSD deteriorations) may strictly increase the mean travel time and this has a negative impact on the VOR. In addition, by contrast with the reliability premium, the VOR is not governed by the absolute reliability proneness function. Indeed, for any random travel time $\tilde{t}_i$ and any two reliability-prone travelers with identical marginal utility of wealth and utility functions u and v over travel time, we get that the VOR of traveler v is greater than the VOR of traveler u if and only if $v(\mu_i) - u(\mu_i) \geq E[v(\tilde{t}_i) - u(\tilde{t}_i)]$ for any $\tilde{t}_i$, which is equivalent to the convexity of $v - u$, that is $v'' \geq u''$ everywhere. Thus, the value of travelers' marginal utility of travel time is not involved.

Once again, consider an alternative i with random travel time $\tilde{t}_i$ expressed as $\mu_i + \tilde{\epsilon}$ where μ_i is the mean travel time and $\tilde{\epsilon}$ is a small actuarially-neutral risk. Thus we get the following approximation of the VOR:[12]

$$VOR_i \approx -\frac{1}{2}\sigma_i^2 \frac{u''(\mu_i)}{\lambda} \approx \Pi_i \times VOT(\mu_i) \tag{20}$$

As a result, we get that the VOR may be expressed as the product of the VOT and of the reliability premium. This makes sense since the VOT may be viewed as a money-metric function which transforms time into money. We can also observe from (20) that the so-called reliability ratio (VOR/VOT) is approximately equal to the reliability premium.

5.0 Empirical implementation of the theory

5.1 Functional forms of travelers' utility

[12] Expanding u around μ_i yields $u(\tilde{t}_i) = u(\mu_i + \tilde{\epsilon}) = u(\mu_i) + \tilde{\epsilon}u'(\mu_i) + \frac{1}{2}\tilde{\epsilon}^2 u''(\mu_i)$. Thus $u(\mu_i) - Eu(\tilde{t}_i) = -\frac{1}{2}\sigma_i^2 u''(\mu_i)$. Finally, using the approximation of the reliability premium in (16) and the approximation of the VOT in (11) with $n = 1$ yields the result.

In empirical studies on the VOT and the VOR, a functional form is given to travelers' preferences function. Therefore, it is of first importance to distinguish between the implications that derive from the modeler's choice of a particular utility function and the ones that derive from empirical facts. We briefly study the three mainly used functional forms: quadratic, negative exponential and power utility functions.

5.11 Quadratic utility functions

Following the theory of finance, the quadratic utility function has been the most widely used in pioneering empirical studies on the VOT and the VOR, such as the one conducted by Jackson and Jucker (1981). The quadratic utility function takes the following form:

$$u(t) = \alpha t + \beta t^2 \tag{21}$$

This function is strictly decreasing and concave if $t > -\alpha/2\beta$ and $\beta < 0$. The absolute reliability proneness function is

$$r(t) = \frac{1}{t+\alpha/2\beta} \tag{22}$$

Thus, travelers with quadratic utility functions exhibit DARP. They would become less reliability-prone for more longer trips. Because the marginal utility of travel time is linear in this case, the third-derivative of the utility function is zero everywhere and, hence, the absolute prudence function is zero, that is $p = 0$ everywhere. Therefore, according to proposition 9, travelers with quadratic utility functions are unaffected by the upside risk. This is so because their preferences function only depends on the first two moments of the distribution of travel time. Substituting (21) in (3), we indeed get a preferences function which only depends on the mean and the variance of travel time:

$$U_i = -\lambda c_i + \alpha \mu_i + \beta \mu_i^2 + \beta \sigma_i^2 \tag{23}$$

This preferences function appears in Senna (1994, eq. 15, p. 211). In Jackson and Jucker (1981), the square of the mean travel time μ_i^2 is omitted.

5.12 Negative exponential utility functions

Other specifications than the quadratic utility function have been used in the transportation literature. In particular, Polak (1987) proposed a negative exponential form:

$$u(t) = -\exp(\beta t) \quad (24)$$

This function has the property that the sign of its nth-derivative is the sign of $-\beta^n$. Thus, if $\beta > 0$, travelers dislike more travel time, are reliability-prone and prudent. In fact, absolute reliability proneness and absolute prudence are equal to the same constant: $r = p = \beta$ everywhere. Thus, the negative exponential form generates CARP (and constant absolute prudence as well). An important consequence of the negative exponential form is that travelers' attitudes towards travel time variability are all unaffected by the trip length.

5.13 Power utility functions

Consider a power utility function known as the Box-Cox transformed:

$$u(t) = \frac{\alpha}{1+\gamma}[t^{1+\gamma} - 1] \quad (25)$$

This function is strictly decreasing if $\alpha < 0$ and concave if $\alpha\gamma \leq 0$. Therefore, if $\alpha < 0$ and $\gamma > 0$, the traveler dislike more travel time and is reliability-prone. If, in addition, $\gamma > 1$, then the traveler is also prudent. On the other hand, if $\alpha < 0$ and $\gamma < 0$, the traveler dislikes more travel time is reliability-averse and (necessarily) prudent.[13] The absolute reliability proneness and prudence functions are:

$$r(t) = \frac{\gamma}{t} \text{ and } p(t) = \frac{\gamma - 1}{t} \quad (26)$$

The main feature of power utility functions is that they generate constant relative reliability proneness. Indeed, it is apparent from (26) that the relative reliability proneness function $r(t)t$ is equal to the constant γ. Therefore, reliability-prone travelers with power utility functions exhibit DARP (and decreasing absolute prudence as well if they are also prudent). Substituting (25) in (3), we get the following preferences function:

[13] The first three successive derivatives of the utility function are: $u'(t)$=αt^{γ}, $u''(t)$=$\alpha\gamma t^{\gamma-1}$ and $u'''(t)$=$\alpha\gamma[\gamma - 1]t^{\gamma-2}$.

$$U_i = -\lambda c_i + \frac{\alpha}{1+\gamma}\left[E\tilde{t}_i^{1+\gamma} - 1\right] \tag{27}$$

In the theory of finance, quadratic utility functions are not in fashion anymore, essentially because they exhibit increasing absolute risk aversion, contradicting the empirical observation that risk premium for additive risks are decreasing with wealth. Thus, power utility functions exhibiting constant relative risk aversion and, hence, decreasing absolute risk aversion, are now the most widely used. In the present context, as a result of the fact that travel time is a non-desirable good, the above point is irrelevant because both quadratic and power utility functions exhibit DARP. However, the power utility function is able to represent the preferences of a wider set travelers than the quadratic utility function. In particular, travelers with quadratic utility functions are indifferent to the upside risk. By contrast, travelers with power utility functions may be prudent or non-prudent as well. Therefore, power utility functions impose less *ad hoc* assumptions about the behavior of real-world travelers and may represent the preferences of a wider set of travelers. Hence, using the power utility function, it is possible to derive explicit analytical formulas of the VOT and the VOR (and of the reliability premium as well) which only depends on parameters λ, α and γ. Substituting (25) in (10), (15) and (18) we get:[14]

$$VOT_i = \frac{-\alpha}{\lambda[1+\gamma]}\left[E\tilde{t}_i^{1+\gamma} - E[\tilde{t}_i - n]^{1+\gamma}\right] \tag{28}$$

$$\Pi_i = \left[E\tilde{t}_i^{1+\gamma}\right]^{\frac{1}{1+\gamma}} - \mu_i \tag{29}$$

and

$$VOR_i = \frac{-\alpha}{\lambda[1+\gamma]}\left[E\tilde{t}_i^{1+\gamma} - \mu_i^{1+\gamma}\right] \tag{30}$$

5.2 Estimations results

To illustrate how our approach can be used to provide behavioral estimations of the VOT and the VOR, we have estimated the parameters of the preferences function (27) using data from a

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[14] We also have the following approximations: $VOT_i \approx -n\frac{\alpha}{\lambda}E[\tilde{t}_i]^{\gamma}$, $\Pi_i \approx \frac{1}{2}\frac{\sigma_i^2}{\mu_i}\gamma$ and $VOR_i \approx -\frac{1}{2}\frac{\sigma_i^2}{\mu_i}\gamma\frac{\alpha}{\lambda}\mu_i^{\gamma}$.

discrete choice experiment in stated preferences framed for personal long distance trips through high-speed rail transportation in France (TGV). The survey was conducted during August and November 2011 and involved 155 voluntary individuals who were offered a series of ten choice sets each, leading to 1550 observations. Each choice set consists in a discrete choice between a fully reliable mode (with a sure travel time) and an unreliable mode (with a random travel time). As in our theoretical model, each mode is fully characterized by only two attributes: its cost represented by the ticket price and the distribution of its travel time. To determine the different levels of attributes, we refer to SNCF's tariffs and travel time indicated for more than two hours trips such as Montpellier-Paris.[15] The estimation results from a conditional logit regression are given in table 1.

Table 1: Conditional logit regression

Cost (λ)	0.072 (0.005)
Reliability attitude (γ)	1.880 (0.062)
Scale parameter (α)	-2.77e-06 (9.01e-07)
Intercept	0.285 (0.085)
Log likelihood (restricted)	-1074.378
Log likelihood (unrestricted)	-460.473
Pseudo R-squared	0.568
Choices predicted with success	78 per cent
Nb. Of observations	1550

The standard deviation of estimated parameters is given in parentheses.

All parameters are significant at the 95 per cent confidence interval.

Once the parameters of the preferences function (27) have been estimated, it is possible to compute the VOT, the reliability premium and the VOR using analytical formulas (28), (29) and (30), respectively. Some illustrative results are given in table 2. The first three alternatives all feature a sure travel time. Hence $\tilde{t}_6$ and $\tilde{t}_7$ are obtained by adding an actuarially-neutral risk $\tilde{\varepsilon}_3$= (-60 min, ½; 60 min, ½) to sure travel times of 180 min and 240 min, respectively.

[15] Precisely, we consider levels of ticket price attribute between 40 € and 150 € and levels of travel time attribute between 120 min and 300 min.

Note also that $\tilde{t}_6$ and $\tilde{t}_7$ may as well be obtained by adding a positive risk $\tilde{\epsilon}_4$= (0 min, ½; 120 min, ½) to sure travel times of 120 min and 180 min, respectively. Finally, $\tilde{t}_8$ and $\tilde{t}_9$ are obtained by adding the actuarially-neutral risk $\tilde{\epsilon}_3$either to the high or to the low travel time realization of $\tilde{t}_6$.

Table 2: Value of time, reliability premium and value of reliability

Alternatives	μ_i	σ_i	VOT_i	Π_i	VOR_i
120 min	120 min	0 min	11 €	0 min	0 €
180 min	180 min	0 min	28 €	0 min	0 €
240 min	240 min	0 min	53 €	0 min	0 €
$\tilde{t}_6$ = (120 min, ½; 240 min, ½)	180 min	60 min	32 €	17 min	12 €
$\tilde{t}_7$ = (180 min, ½; 300 min, ½)	240 min	60 min	57 €	13 min	16 €
$\tilde{t}_8$ = (120 min, ½; 180 min, ¼, 300 min, ¼)	180 min	73 min	34 €	27 min	21 €
$\tilde{t}_9$ = (60 min, ¼; 180 min, ¼; 240 min, ½)	180 min	73 min	34 €	23 min	17 €

The VOT is calculated for one hour of travel time saved for sure (n = 60 min).

Because the estimated parameter γ of the power utility function is positive and greater than one, the representative traveler for our sample is both reliability-prone and prudent. As demonstrated in proposition 12, the VOT is therefore increasing with the travel time. Besides, because the representative traveler is prudent, the VOT is also convex. Indeed, we observe in table 2 that the VOT increases from 11 € to 28 € as the travel time increases from 120 min to 180 min while the VOT increases from 28 € to 53 € as the travel time increases from 180 min to 240 min. Furthermore, as demonstrated in propositions 13 and 14, we observe that the VOT is increasing with the variability of travel time. For instance, the VOT increases from 32 € to 34 € when $\tilde{t}_6$ is affected by the actuarially-neutral risk $\tilde{\epsilon}_3$ (leading to $\tilde{t}_8$ and $\tilde{t}_9$). Of course, as demonstrated in propositions 16 and 19, this is also the case of the reliability premium and of the VOR.

Moreover, because the power utility function exhibits DARP, we observe that the reliability premium is decreasing with the mean travel time of the alternative. For instance, the reliability premium decreases from 17 min to 13 min when 60 min are added for sure to

$\tilde{t}_6$ (leading to $\tilde{t}_7$). Thus, under DARP, the reliability premium is smaller for more longer trips. Nevertheless, because the VOT is increasing with travel time, the VOR is, by contrast with the reliability premium, increasing with the travel time. Indeed, by comparing $\tilde{t}_6$ and $\tilde{t}_7$, we observe that the VOR increases from 12 € to 16 €.

6.0 Conclusion

The paper dealt with travelers' willingness to pay to save travel time and to improve its reliability. We derived an intuitive theoretical model of choice among transport alternatives characterized by their price and the distribution of their travel time. The VOT and the VOR have been precisely defined and theirs properties have been established. Finally, we have used data from a discrete choice experiment in stated preferences to illustrate how our approach could be easily implemented to provide behavioral estimations of the VOT and the VOR. The VOT is between 11 € and 57 € for one hour of travel time saved and the VOR is between 12 € and 16 € for one hour of standard deviation of travel time saved. Even if our empirical application is only illustrative, we obtain very plausible values of the VOT and the VOR which are in line with the estimations reported in the literature (for example Zamparini and al. (2007), Abrantes and Wardman (2011) and Devarasetty and al. (2012).). In practice, the reference VOT used by French public authorities (Quinet, 2013) in railway investment projects is between 12 € and 44 € for one hour of travel time saved.

7.0 Appendix

7.1 Proof of proposition 1

(ii)⇒*(i)*. Define $\delta(t) = F_i(t) - F_j(t)$ and observe that $\delta\left(t^{min}\right) = \delta(t^{max}) = 0$. By definition, *(ii)* is equivalent to $\delta \geq 0$ everywhere, while *(i)* is equivalent to $u_i - u_j = \int_{t^{min}}^{t^{max}} u(t)d\delta(t) \geq 0$ for all u with $u' \leq 0$ everywhere. Integrating by parts, we get $u_i - u_j = -\int_{t^{min}}^{t^{max}} u'(t)\delta(t)dt$. It is then clear that *(ii)* is sufficient for *(i)*.

(i) ⇒*(ii)*. We prove the necessity of *(ii)* by contradiction. Suppose that *(ii)* is not true, so that $\delta(t^*)$< 0 for some t^*. Hence, we can always find a decreasing utility function u such that *(i)* would be violated. For instance, take $u(t)$ = k> 0 for $t \leq t^*$ and $u(t)$ = 0 for $t > t^*$. With this utility function $u_i - u_j$ = $k \int_{t^{min}}^{t^*} d\,\delta(t)$ = $k\delta(t^*)$< 0, which contradicts *(i)*. ■

7.2 Proof of proposition 3

(iii)⇔*(iv)*. By definition, *(iii)* is equivalent to $u(E\tilde{t}) \geq Eu(\tilde{t})$ for any $\tilde{t}$. From Jensen (1906)'s inequality, we know that this is true if and only if u is concave over the support of $\tilde{t}$. ■

7.3 Proof of proposition 4

(vi) ⇒*(v)*. Define $\Delta(t)$ = $\int_{t}^{t^{max}} \delta(s)ds$ and observe that $\Delta(t^{max})$ = 0. Thus, *(vi)* is equivalent to $\Delta \geq 0$ everywhere with μ_i = μ_j. Then note that $\Delta\left(t^{min}\right)$= 0 is equivalent to μ_i = μ_j. Indeed, by definition, $\Delta\left(t^{min}\right)$ = $\int_{t^{min}}^{t^{max}} \delta(t)dt$ = 0, while $\mu_i - \mu_j$ = $\int_{t^{min}}^{t^{max}} t d\delta(t)$. Integrating by parts, we get $\mu_i - \mu_j$ = $-\Delta\left(t^{min}\right)$. Thus, *(vi)* is equivalent to $\Delta \geq 0$ everywhere with $\Delta\left(t^{min}\right)$ = 0, while *(v)* is equivalent to $u_i - u_j$ = $\int_{t^{min}}^{t^{max}} u(t)d\delta(t) \geq 0$ for all u with $u'' \leq 0$ everywhere. Integrating by parts twice we get $u_i - u_j$ = $-\int_{t^{min}}^{t^{max}} u''(t)\Delta(t)dt$. It is then clear that *(vi)* implies *(v)*.

(v) ⇒*(vi)*. We prove the necessity of *(vi)* by contradiction. Suppose that *(vi)* is not true, so that $\Delta(t^*)$< 0 for some t^*. In this context, we can always find a utility function u which is linear above and below t^* and strictly concave in the neighborhood of t^*, such that *(v)* would be violated. ■

7.4 Proof of proposition 5

(vii)⇔*(viii)*. We have proved in proposition 1 that all travelers who dislike more travel time prefer $\tilde{t}_i$ to $\tilde{t}_j$ if and only if $\tilde{t}_j$ is a FSD deterioration of $\tilde{t}_i$. Besides, we have proved in proposition 4 that all reliability-prone travelers prefer $\tilde{t}_i$ to $\tilde{t}_j$ if and only if $\tilde{t}_j$ contains more

actuarially-neutral risk than $\tilde{t}_i$. Hence, the proof of proposition 5 follows by observing that any SSD deterioration can be obtained by combining any FSD deterioration with any actuarially-neutral increase in risk. ■

7.5 Proof of proposition 7

(x)⇔*(xi)*. Taking as given that the utility functions u and v are decreasing, there always exists an increasing function $\psi : \mathbb{R} \to \mathbb{R}$ defined as $\psi(u) = v$. Thus, *(xi)* is equivalent to $\psi'' \leq 0$ everywhere. By chain rule, we get $v' = \psi'(u)u'$ and $v'' = \psi''(u)[u']^2 + \psi'(u)u''$. Combining these two relation yields $v''/v' - u''/u' = u'(t)\psi''(u(t))/\psi'(u)$. Thus *(x)* is equivalent to *(xi)*.

(xi)⇒*(ix)*. Consider a random travel time expressed as $t + \tilde{\epsilon}$, where t is a positive constant and $\tilde{\epsilon}$ is any risk that is disliked by traveler u, that is any risk $\tilde{\epsilon}$ satisfying $Eu(t + \tilde{\epsilon}) \leq u(t)$. According to definition 6, *(ix)* is equivalent to $Eu(t + \tilde{\epsilon}) \leq u(t) \Rightarrow Ev(t + \tilde{\epsilon}) \leq v(t)$. Substituting $\psi(u) = v$, *(ix)* is also equivalent to $Eu(t + \tilde{\epsilon}) \leq u(t) \Rightarrow E\psi(u(t + \tilde{\epsilon})) \leq \psi(u(t))$. On the other hand, by Jensen (1906)'s inequality, *(xi)* is equivalent to $E\psi(u(t + \tilde{\epsilon})) \leq \psi(Eu(t + \tilde{\epsilon}))$. Because ψ is increasing, we also have $Eu(t + \tilde{\epsilon}) \leq u(t) \Rightarrow \psi\left(E(u(t + \tilde{\epsilon}))\right) \leq \psi(u(t))$. Thus, *(xi)* implies that $Eu(t + \tilde{\epsilon}) \leq u(t) \Rightarrow E\psi(u(t + \tilde{\epsilon})) \leq \psi\left(E(u(t + \tilde{\epsilon}))\right) \leq \psi(u(t))$.

(ix)⇒*(xi)*. We prove the reverse by contradiction. Suppose that *(xi)* is not true so that there exists an interval in the image of u where $\psi'' > 0$. Suppose also that the random travel time $t + \tilde{\epsilon}$ has its support on that interval and a utility function u satisfying $Eu(t + \tilde{\epsilon}) = u(t)$. By Jensen (1906)'s inequality, $\psi'' > 0$ implies $E\psi(u(t + \tilde{\epsilon})) > \psi(Eu(t + \tilde{\epsilon}))$ or, equivalently, $Ev(t + \tilde{\epsilon}) > v(t)$, which contradict *(ix)*. ■

7.6 Proof of proposition 9

(xiii)⇒*(xii)*. Define $\Theta(t) = \int_t^{t^{max}} \Delta(s)ds$ and observe that $\Theta(t^{max}) = 0$. Thus, *(xiii)* is equivalent to $\Theta \geq 0$ everywhere with $\mu_i = \mu_j$ and $\sigma_i^2 = \sigma_j^2$. Recall that $\mu_i = \mu_j$ is equivalent to $\Delta(t^{min}) = 0$ (see the proof of proposition 4). Furthermore, if $\mu_i = \mu_j$, then $\sigma_i^2 - \sigma_j^2 = \int_{t^{min}}^{t^{max}} t^2 d\delta(t)$. Integrating by parts twice, we get $\sigma_i^2 - \sigma_j^2 = -2\int_{t^{min}}^{t^{max}} t d\delta(t) = -2\Theta(t^{min})$. Thus, $\sigma_i^2 = \sigma_j^2$ is equivalent to $\Theta(t^{min}) = 0$. Hence, *(xiii)* is equivalent to $\Theta \geq 0$ everywhere with $\Delta(t^{min}) = \Theta(t^{min}) = 0$, while *(xii)* is equivalent to $u_i - u_j = \int_{t^{min}}^{t^{max}} u(t)d\delta(t)dt \geq 0$ for all u with $u''' \leq 0$ everywhere. Integrating by parts three times we get $u_i - u_j = -\int_{t^{min}}^{t^{max}} u'''(t)\Theta(t)dt$. It is then clear that *(xiii)* is sufficient for *(xii)*.

(xii)⇒*(xiii)*. We prove the necessity of *(xiii)* by contradiction. Suppose that *(xiii)* is not true, so that $\Theta(t^*) < 0$ for some t^*. In this context, we can always find a utility function u with its slope being linear above and below t^* and strictly concave in the neighborhood of t^*, such that *(xii)* would be violated. ■

7.7 Proof of proposition 10

(xiv)⇔*(xv)*. We have already proved in proposition 5 that all reliability-prone travelers who dislike more travel time prefer $\tilde{t}_i$ to $\tilde{t}_j$ if and only if $\tilde{t}_j$ is a SSD deterioration of $\tilde{t}_i$. In addition, we have proved in proposition 9 that all prudent travelers prefer $\tilde{t}_i$ to $\tilde{t}_j$ if and only if $\tilde{t}_j$ contains more upside risk than $\tilde{t}_i$. Hence, proposition 10 directly follows from these two results by observing that any TSD deterioration can be obtained by combining any SSD deterioration with any increase in upside risk. ■

7.8 Proof of proposition 12

(ii)⇒*(xvi)*. By definition, *(ii)* is equivalent to $\delta \geq 0$ everywhere (see the proof of proposition 1), while *(xvi)* is equivalent to $VOT_i - VOT_j = \frac{1}{\lambda}\int_{t^{min}}^{t^{max}} [u(t-n) - u(t)]d\delta(t) \leq 0$ for all u with $u'' \leq 0$ everywhere. Integrating by parts, we get $VOT_i - VOT_j = -\frac{1}{\lambda}\int_{t^{min}}^{t^{max}} [u'(t-n) -$

$u'(t)]\delta(t)dt$. Because $\lambda > 0$ and $n > 0$, it is clear that $VOT_j \geq VOT_i$ for all u with $u'' \leq 0$ everywhere. Thus *(ii)* is sufficient for *(xvi)*.

(xvi)⇒(ii). We prove the necessity of *(ii)* by contradiction. Suppose that *(ii)* is not true, so that $\delta(t^*)< 0$ for some t^*. In this context, we can always find a utility function u which is linear above and below t^* and strictly concave in the neighborhood of t^*, such that *(xvi)* would be violated. ■

7.9 Proof of proposition 13

(vi)⇒(xvii). By definition, *(vi)* is equivalent to $\Delta \geq 0$ everywhere with $\Delta(t^{min}) = 0$ (see the proof of proposition 4), while *(xvii)* is equivalent to $VOT_i - VOT_j = \frac{1}{\lambda}\int_{t^{min}}^{t^{max}}[u(t-n) - u(t)]d\delta(t) \leq 0$ for all u with $u''' \leq 0$ everywhere. Integrating by parts twice, we get $VOT_i - VOT_j = -\frac{1}{\lambda}\int_{t^{min}}^{t^{max}}[u''(t-n) - u''(t)]\Delta(t)dt$. Because $\lambda > 0$ and $n > 0$, it is clear that *(vi)* implies *(xvii)*.

(xvii)⇒(vi). We prove the necessity of *(vi)* by contradiction. Suppose that *(vi)* is not true, so that $\Delta(t^*)< 0$ for some t^*. In this context, we can always find a utility function u which is linear above and below t^* and non-linear with a strictly negative third-derivative in the neighborhood of t^*, such that *(xvii)* would be violated. ■

7.10 Proof of proposition 14

(xviii)⇔(viii). We have already proved in proposition 12 that for all reliability-prone travelers, the VOT is greater for $\tilde{t}_i$ than for $\tilde{t}_j$ if and only if $\tilde{t}_j$ is a FSD deterioration of $\tilde{t}_i$. In addition, we have proved in proposition 13 that for all prudent travelers, the VOT is greater for $\tilde{t}_i$ than for $\tilde{t}_j$ if and only if $\tilde{t}_j$ contains more actuarially-neutral risk than $\tilde{t}_i$. Hence, proposition 14 is proved by observing that any SSD deterioration can be obtained by combining any FSD deterioration with any increase in actuarially-neutral risk. ■

7.11 Proof of proposition 16

(vi)⇔(xix). We know from proposition 4 that all reliability-prone travelers prefer $\tilde{t}_j$ to$\tilde{t}_i$ if and only if *(vi)* is true. Thus *(vi)* is equivalent to $u_i - u_j = u(\mu_i + \Pi_i) - u(\mu_j + \Pi_j) \geq 0$ for all u with $u'' \leq 0$ everywhere. Using the fact that $\mu_i = \mu_j = \mu$, *(vi)* is also equivalent to $u(\mu + \Pi_i) \geq u(\mu + \Pi_j)$ for all u with $u'' \leq 0$ everywhere. Thus *(vi)* is equivalent to $\Pi_i \leq \Pi_j$ for all u with $u' < 0$ and$u'' \leq 0$ everywhere. ■

7.12 Proof of proposition 17

(xx) ⇔ *(ix)*. Whenever $v' < 0$ everywhere, *(xx)* is equivalent to $Eu(\tilde{t}_i) = u(\mu_i + \Pi_i) \Rightarrow Ev(\tilde{t}_i) \leq v(\mu_i + \Pi_i)$. The l.h.s. of the implication simply says that Π_i is the reliability premium of traveler u. The r.h.s. of the implication says that the reliability premium of traveler v is greater than the one of traveler u. Because any random travel time $\tilde{t}_i$ may be written as $\mu_i + \tilde{\epsilon}$, where $\tilde{\epsilon}$ is an actuarially-neutral risk, *(xx)* is equivalent to $Eu(\mu_i + \tilde{\epsilon}) = u(\mu_i + \Pi_i) \Rightarrow Ev(\mu_i + \tilde{\epsilon}) \leq v(\mu_i + \Pi_i)$. Defining $t = \mu_i + \Pi_i$ and $\tilde{\tilde{\varepsilon}} = \tilde{\epsilon} - \Pi_i$, *(xx)* may be rewritten as $Eu(t + \tilde{\tilde{\varepsilon}}) = u(t) \Rightarrow Ev(t + \tilde{\tilde{\varepsilon}}) \leq v(t)$. This is equivalent to say that traveler v rejects all risks affecting a sure travel time t for which u is indifferent. We know from proposition 7 that this is true if and only if *(ix)* is true. ■

7.13 Proof of proposition 19

(vi)⇒(xxi). By definition, *(vi)* is equivalent to $\Delta \geq 0$ everywhere with $\Delta(t^{min}) = 0$ which is equivalent to $\mu_i = \mu_j$ (see the proof of proposition 4), while *(xxi)* is equivalent to $VOR_i - VOR_j = -\frac{1}{\lambda}\int_{t^{min}}^{t^{max}} u(t)d\delta(t) \leq 0$ for all for all u with $u'' \leq 0$ everywhere. Taking as given that $\tilde{t}_i$ and $\tilde{t}_j$ have the same mean and integrating by parts twice we get $VOR_i - VOR_j = \int_{t^{min}}^{t^{max}} u''(t)\Delta(t)dt$. It is then clear that *(vi)* is sufficient for *(xxi)*.

(xxi)⇒(vi). We prove the necessity of *(vi)* by contradiction. Suppose that *(vi)* is not true, so that $\Delta(t^*) < 0$ for some t^*. In this context, we can always find a utility function u which is

linear above and below t^* and strictly concave in the neighborhood of t^*, such that *(xxi)* would be violated. ■

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