«Addressing agent specific extreme price risk in the presence of heterogeneous data sources: A food safety perspective»

Abdoul Salam DIALLO
Alfred MBAIRADJIM MOUSSA

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Addressing agent specific extreme price risk in the presence of heterogeneous data sources: A food safety perspective

A. S. Diallo∗ A. Mbairadjim Moussa
LAMETA LAMETA
Université Montpellier I Université Montpellier I

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Abstract

Since the recent food crisis, there has been prominent literature relating to the study of commodity prices volatility and other factors impacting on food security and resilience measures, especially in an African context. Numerous studies have shown that aside production factors, commodities prices instability and the associated price anticipation difficulties constitute a major point of preoccupation when it comes to food insecurity assessment. The multiplicity of historical price data represents one of such difficulties, as it creates uncertainty when making investment decisions.

This paper assumes that risk factors based on these historical prices are observed under uncertainty with no unique observable efficient price, and are therefore modeled as interval-valued random variables. Under such conditions, a linear portfolio of agricultural commodities characterized by a vector of their interval-valued risk factors is considered, just as extreme risk models (VaR and ES) under uncertainty as introduced by Mbairadjim Moussa et al. (2014) are applied for risk computation and portfolio selection. Moreover, our paper proposes specific portfolio selection models under uncertainty, which discriminate between producers, consumers and political decision-makers concerns, in establishing an optimally allocated portfolio.

Finally some empirical examples carried out on real data set from Niger, show the effectiveness of the proposed methods in a practical decision making context

Key-words: Portfolio optimization; Value-at-risk; Expected Shortfall; Knightian uncertainty; Agricultural commodities; African market; Random closed sets.

JEL Codes: C6, D8, G1.

∗Corresponding author. LAMETA Université de Montpellier I, UFR d’Economie Avenue Raymond DUGRAND - Site de Richter C.S. 79606 34960 MONTPELLIER CEDEX 2 France, Courriel: mbairadjim@lameta.univ-montp1.fr, moussa_alf@yahoo.fr
1 Introduction

Since the recent food crisis over the last decade, much attention have been drawn to factors impacting on food security and resilience measures, particularly in an African context. It has been quite well established that, aside production factors, the most worrisome aspect of food insecurity lies with commodities prices instability and the associated price anticipation difficulties. Thus the very prominent literature relating to the study of commodity prices volatility.

Amongst the reasons advanced in order to explain such price instabilities, we can cite the World Bank’s (2007) report linking them to a long tradition of agricultural sector underfunding. Other authors such as Abbot et al. (2008) or Mitchell (2008) incriminate the new bio-fuel policies, a remarkably depreciated dollar and historically low stock levels. More recently, speculation has also been shown to play an important role in rendering prices unstable (Cooke and Robles (2009)).

Commodity prices instability is such that, both its ascending and descending trends are problematic when extreme. Indeed, at a micro-economic level, abrupt price hikes result in higher poverty and an acute marginalization of vulnerable households from the food system. On another hand, severe price falls will induce losses suffered by farmers who are faced with revenues lower than their production costs. It is noteworthy that the dominant type of farming structure in developing areas is family farming where outputs are meant not only for consumption by the producing entity but also for trade. In this particular case, both facets of price evolution impact directly on the considered household.

Considered under an African perspective, the notion of food insecurity due to price instability is not only present, but also takes an additional form that translates into price multiplicity on local food markets. Indeed, agricultural commodities markets in the Sub-Saharan region are characterized by the fact that prices applied on them are often the result of a bargain between buyers and sellers, which grants these prices a variable nature. Thus even in the same market place, different buyers can be offered different prices for an identical commodity, by the same trader and in the same time period. This calls into question the hypothesis relating to the existence of an efficient price.

Furthermore, this price variability of a commodity at a given transaction time is also observed for different market places of the same city. The variable aspect of prices applied on these markets results into ambiguity and/or imprecision in the identification of an “efficient” price. Therefore, when representing agents’ preferences using historical price data, a unique probability distribution would not be sufficient, because the so-identified imprecision adds onto the stochastic variability of prices.
Beyond consumers and farmers, decision-makers (in charge of guaranteeing food security in these areas) are also concerned with this issue, since it disrupts their ability to properly track safeguard needs on their domestic markets. To aid them in their task, we suggest that the political decision-maker resorts to portfolio theory tools in order to optimally allocate investments in the various commodities making up the basic food basket, to ensure food insecurity resilience. Since the seminal work of Markowitz (1952), the modern portfolio selection theory has been one of the most active areas of quantitative finance. It is concerned with selecting a combination of securities from a portfolio containing large numbers of securities, in order to reach an investment goal. The optimal combination is obtained in practice by minimizing a risk measure for a given minimum expected return. The primary approach of Markowitz, using the variance of returns as the risk measure, is subjected to the assumption of normal distribution of returns. However, financial returns being generally leptokurtic and asymmetric, the use of only mean value and variance for portfolio selection becomes inadequate. For this reason, the Markowitz (1952) model was improved in various ways by the use of more elaborate risk measures such as higher moments (Arditti, 1967; Samuelson, 1970; Kraus and Litzenberger, 1976; Konno et al., 1993; Konno and Suzuki, 1995), partial moments (Estrada, 2008; Fishburn, 1977; Markowitz et al., 1993; and Konno et al., 2002), Value-at-risk or Expected shortfall (Bertsimas et al., 2004). In practice, these models are implemented using historical data-sets of returns, referred to as risk factors, and assuming that there exists a unique and efficient prices process. Estimation of the risk measures is based on straightforward descriptive statistics. Hence a unique probability distribution is used in modeling their stochastic variability.

However, our decision-maker is confronted to inaccurate knowledge about the state of nature. Indeed, on some markets, like those of agricultural commodities in the African sub-region, the existence of an efficient price is not always given. Not only the prices applied on these markets are usually the outcome of a bargaining process, but also, for a given commodity, prices may display a significant discrepancy from one market to another or from one sales person to another within the same market. An additional explanation for this matter is the fact that the price of a commodity may vary when dealing with its retail or wholesale version, while some "agents" may display the double function of wholesalers or retailers. Furthermore, the practice of a wholesale or retail price is strictly dependent upon the broker’s will, thus an agents willing to operate on such markets does not exactly know what price he would be confronted to. When representing the agent’s preferences using historical data of these prices, a unique probability distribution would not be sufficient.

In this setting, the political decision maker or investor who operates on these markets is said to face Knightian uncertainty, due to the combination of stochastic variability of the risk factor and the imprecision and/or ambiguity in the observation of the efficient price. His behavior contradicts with the Von Neumann-Morgenstern paradigm as well as that of Savage (1954), which assumes the existence of a unique well-defined additive probability distribution
that represents the beliefs of an individual.

In this paper, we apply extreme risk assessment measures under knightian uncertainty from portfolio theory to food insecurity management. This modeling approach allows us to take into account stochastic variability and imprecision of risk factors that affect their observation. The Value-at-Risk and Expected Shortfall models with interval-valued risk factors of Mbairadjim Moussa et al. (2014) are used for extreme risk measurement of an agricultural commodities portfolio. These risk measures are then applied for optimal decision making in a food safety perspective.

The remainder of this paper is organized as follows: Section 2 presents the concepts of extreme risk measures with interval-valued risk factors. Section 3 describes the portfolio selection model developed to reach our goal. Finally, section 4 covers empirical analysis and applications while section 5 concludes.

2 Extreme risk measurement under interval-valued risk factors

In this section, we briefly presents the VaR and ES models with interval-valued risk factors of Mbairadjim Moussa et al. (2014). We also explain how these risk factors are computed in our case from multiple historical prices time series of a given commodity.

2.1 VaR and ES model with interval-valued risk factors

Let $\tilde{\Pi}(t)$ be a linear portfolio with interval-valued price (or value) at time $t$ with $n$ components. The relative change of this value in a time window $[0, t]$, which is a random closed interval, is defined by a linear combination (in the Minkowski sens), of its interval-valued risk factors

$$
\Delta \tilde{\Pi}(t) = \delta_1 \tilde{X}_1 + \delta_2 \tilde{X}_2 + ... + \delta_n \tilde{X}_n,
$$

with $\tilde{X}_1, ..., \tilde{X}_n$ being the interval-valued risk factors of its constituents over the same period and $\delta = (\delta_1, \delta_2, ..., \delta_n)$ the real portfolio weights vector such that $\delta_1 + \delta_2 + ... + \delta_n = 1$ and $\delta_i \geq 0, \forall i = 1, ..., n$. As we only deal with the period $[0, t]$, the index $t$ will be omitted from formal expressions in the remainder of the paper.

We call on the following definitions:

**Definition 2.1** Let $\Delta \Pi$ be the interval-valued risk factor of the portfolio $\Pi$ at time $t$ such that its lower and upper bounds are the random variables $\Delta \Pi^l$ and $\Delta \Pi^u$. The corresponding Value-at-risk couple at confidence level $1 - \theta$, $VaR_\theta = \{VaR_\theta(\Delta \Pi^l), VaR_\theta(\Delta \Pi^u)\}$ is defined by:

$$
P\{\Delta \Pi^l < -VaR_\theta(\Delta \Pi^l)\} = \theta \quad (2)$$

$$
P\{\Delta \Pi^u < -VaR_\theta(\Delta \Pi^u)\} = \theta \quad (3)$$
Definition 2.2 Let $\Delta \tilde{\Pi}(t)$ be the interval-valued risk factor of the portfolio $\Pi$ at time $t$ such that its lower and upper bounds are the random variables $\Delta \Pi^l$ and $\Delta \Pi^u$. The corresponding Expected shortfall couple at confidence level $1 - \theta$, $ES_\theta = \{ ES_\theta(\Delta \Pi^l), ES_\theta(\Delta \Pi^u) \}$ is defined by:

\[
es_\theta(\Delta \Pi^l) = \mathbb{E}[ -\Delta \Pi^l | -\Delta \Pi^l > -VaR_\theta(\Delta \Pi^l) ] \tag{4}\]
\[
es_\theta(\Delta \Pi^u) = \mathbb{E}[ -\Delta \Pi^u | -\Delta \Pi^u > -VaR_\theta(\Delta \Pi^u) ] \tag{5}\]

Remark that in definitions 2.2 and 2.1, the elements of the interval-valued VaR (resp. ES) couple are the VaRs (resp. ES) of the upper and lower bounds of the interval-valued portfolio risk factor. In the two cases, these elements can be interpreted respectively as the "producer-relevant" and "consumer-relevant" values of the risk measure. The investors referred to here are considered to be uncertainty and risk averse. They will always expect the "worse case scenario" to come to pass, and therefore elaborate hedging strategies against associated outcomes. Said "worse case scenario" will correspond to low price values for farmers and high prices for consumers.

We present some analytical expressions of these risk measures under the assumption of normal and Student distributions of the interval-valued risk factors bounds. The proof of these results is available in Mbairadjim Moussa et al. (2014).

The random vectors of the interval-valued risk factors are denoted $\tilde{X} = (\tilde{X}_1, ..., \tilde{X}_n)$. Since each risk factor is characterized by its bounds, $\tilde{X}_i = [X^l_i, X^u_i]$ ($i = 1, ..., n$), we consequently redefine the random vectors of risk factors as represented by two real random vectors,

\[
X^l = (X^l_1, ..., X^l_n), \quad X^u = (X^u_1, ..., X^u_n). \tag{6}\]

The weights $\delta_i$ ($i = 1, ..., n$) are all positive, it follows that upper and the lower bounds of the portfolio risk factor can be expressed by:

\[
\Delta \Pi^l = \sum_{i=1}^{n} \delta_i X^l_i, \quad \Delta \Pi^u = \sum_{i=1}^{n} \delta_i X^u_i. \tag{7}\]

2.1.1 VaR and ES under gaussian distributions

The random vectors of lower and upper bounds are assumed to be normally distributed and written as

\[
X^l \sim \mathcal{N}(\mu^l, \Sigma^l), \quad X^u \sim \mathcal{N}(\mu^u, \Sigma^u); \tag{8}\]

We present some analytical expressions of these risk measures under the assumption of normal and Student distributions of the interval-valued risk factors bounds. The proof of these results is available in Mbairadjim Moussa et al. (2014).
where \( \mu^k \) is the mean vector, \( \Sigma^k \) the variance-covariance matrix (\( k = l, u \)).

We have the following results:

**Theorem 2.3** Let \( \Delta \Pi = \delta_1 \tilde{X}_1 + ... + \delta_n \tilde{X}_n \) be the interval-valued return of the portfolio, with risk factors \( \tilde{X}_i = (X^l_i, X^u_i) \), for \( i = 1, ..., n \). If we assume that \( (X^l_1, ..., X^l_n) \sim N(\mu^l, \Sigma^l) \) and \( (X^u_1, ..., X^u_n) \sim N(\mu^u, \Sigma^u) \), then the corresponding Value-at-risk couple at confidence level \( 1 - \theta \), \( VaR_{\theta} = \{ VaR_{\theta}(\Delta \Pi^l), VaR_{\theta}(\Delta \Pi^u) \} \) is given by

\[
VaR_{\theta}(\Delta \Pi^l) = \sum_{i=1}^{n} \delta_i \mu^l_i + q_{1-\theta} \sqrt{\sum_{j=1}^{n} \sum_{i=1}^{n} \delta_i \delta_j \sigma^l_{ij}},
\]

\[
VaR_{\theta}(\Delta \Pi^u) = \sum_{i=1}^{n} \delta_i \mu^u_i + q_{1-\theta} \sqrt{\sum_{j=1}^{n} \sum_{i=1}^{n} \delta_i \delta_j \sigma^u_{ij}};
\]

where \( q_{1-\theta} \) is the quantile of level \( 1 - \theta \) of the standard normal distribution.

**Theorem 2.4** Let \( \Delta \Pi = \delta_1 \tilde{X}_1 + ... + \delta_n \tilde{X}_n \) be the interval-valued return of the portfolio, with risk factors \( \tilde{X}_i = (X^l_i, X^u_i) \), for \( i = 1, ..., n \). If we assume that \( (X^l_1, ..., X^l_n) \sim N(\mu^l, \Sigma^l) \) and \( (X^u_1, ..., X^u_n) \sim N(\mu^u, \Sigma^u) \), then the corresponding Expected shortfall couple at confidence level \( 1 - \theta \), \( ES_{\theta} = \{ ES_{\theta}(\Delta \Pi^l), ES_{\theta}(\Delta \Pi^u) \} \) is given by

\[
ES_{\theta}(\Delta \Pi^l) = \sum_{i=1}^{n} \delta_i \mu^l_i + \kappa_\theta \sqrt{\sum_{j=1}^{n} \sum_{i=1}^{n} \delta_i \delta_j \sigma^l_{ij}},
\]

\[
ES_{\theta}(\Delta \Pi^u) = \sum_{i=1}^{n} \delta_i \mu^u_i + \kappa_\theta \sqrt{\sum_{j=1}^{n} \sum_{i=1}^{n} \delta_i \delta_j \sigma^u_{ij}};
\]

where \( \kappa_\theta = \frac{1}{\theta \sqrt{2\pi}} \exp \left( -\frac{(q_\theta)^2}{2} \right) \) and \( q_\theta \) the quantile of level \( \theta \) of the standard normal distribution.

2.1.2 VaR and ES under Student distributions

The random vectors of lower and upper bounds are assumed to have Student distribution and written as

\[
X^l \sim T(\mu^l, \Sigma^l, \nu^l), \quad X^u \sim T(\mu^u, \Sigma^u, \nu^u);
\]

where \( \mu^k \) is the mean vector, \( \Sigma^k \) the variance-covariance matrix and \( \nu^k \) the degree of freedom (\( k = l, u \)).

**Theorem 2.5** Let \( \Delta \Pi = \delta_1 \tilde{X}_1 + ... + \delta_n \tilde{X}_n \) be the interval-valued return of the portfolio, with risk factors \( \tilde{X}_i = (X^l_i, X^u_i) \), for \( i = 1, ..., n \). If we assume that \( (X^l_1, ..., X^l_n) \sim T(\mu^l, \Sigma^l, \nu^l) \) and
\[(X_1, ..., X_n) \sim \mathcal{T}(\mu^*, \Sigma^*, \nu^*),\] then the corresponding Value-at-risk couple at confidence level \(1 - \theta,\)
\[\text{VaR}_\theta = \{\text{VaR}_\theta(\Delta \Pi^I_\alpha), \text{VaR}_\theta(\Delta \Pi^u_\alpha)\}\] is given by
\[
\text{VaR}_\theta(\Delta \Pi^I_\alpha) = \sum_{i=1}^{n} \delta_i \mu_I^l + q_{\theta,1-\theta} \sqrt{\sum_{j=1}^{n} \sum_{i=1}^{n} \delta_i \delta_j \sigma_{ij}^l}, (14) \\
\text{VaR}_\theta(\Delta \Pi^u_\alpha) = \sum_{i=1}^{n} \delta_i \mu_u^l + q_{\theta,1-\theta} \sqrt{\sum_{j=1}^{n} \sum_{i=1}^{n} \delta_i \delta_j \sigma_{ij}^u}; (15) \\
\]
where \(q_{\theta,1-\theta}\) and \(q_{\theta,1-\theta}\) are the quantiles of level \(1 - \theta\) of standard t-Student distributions with respectively \(\nu^l\) and \(\nu^u\) degrees of freedom.

**Theorem 2.6** Let \(\Delta \Pi = \delta_1 \tilde{X}_1 + ... + \delta_n \tilde{X}_n\) be the interval-valued return of the portfolio, with risk factors \(\tilde{X}_i = (X_i^l, X_i^u),\) for \(i = 1, ..., n.\) If we assume that \((X_1^l, ..., X_n^l) \sim \mathcal{T}(\mu^l, \Sigma^l, \nu^l)\) and \((X_1^u, ..., X_n^u) \sim \mathcal{T}(\mu^u, \Sigma^u, \nu^u),\) then the corresponding Expected shortfall couple at confidence level \(1 - \theta, ES_\theta = \{ES_\theta(\Delta \Pi^I_\alpha), ES_\theta(\Delta \Pi^u_\alpha)\}\) is given by
\[
ES_\theta(\Delta \Pi^I_\alpha) = \sum_{i=1}^{n} \delta_i \mu_I^l + \kappa_\theta \sqrt{\sum_{j=1}^{n} \sum_{i=1}^{n} \delta_i \delta_j \sigma_{ij}^l}; (16) \\
ES_\theta(\Delta \Pi^u_\alpha) = \sum_{i=1}^{n} \delta_i \mu_u^l + \kappa_\theta \sqrt{\sum_{j=1}^{n} \sum_{i=1}^{n} \delta_i \delta_j \sigma_{ij}^u}; (17) \\
\]
where \(\kappa_{\theta,1/2} = \frac{1}{\theta \sqrt{\pi}} \Gamma((\nu^k-1)/2) \nu^k \nu^k/2 \left((q_{\theta,1/2})^2 + \nu^k\right)^{-(\nu^k-1)/2},\) with \(k = l, u\) and \(q_{\theta,1/2} \) and \(q_{\theta,1/2}\) the quantiles of level \(\theta\) of standard t-Student distributions with respectively \(\nu^l\) and \(\nu^u\) degrees of freedom.

### 2.2 Risk factors representation

In the African context of commodity markets, for reasons exposed previously, there exists at a given time, various prices for the same underlying asset. Hence, investors are confronted to the existence of multiple historical price data. The price differences thus observed being non negligible, a given investor has to consider separately each of these data. Assume that the investor has \(K\) sources of information which gives prices time series \(P^j_{i,t} = \{P^j_{i,t,1}, ..., P^j_{i,t,T}\},\) \(i = 1, ..., K\) of a commodity \(j = 1, ..., n.\) We denote by \(P^j = \{P^j_{1,t}, ..., P^j_{T,t}\}\) the time series of the efficient price of commodity \(j\) which is not observed by the investors and we adopt the following assumption

**Assumption 2.7** The unobserved efficient price for a commodity \(i\) at any time \(t = 1, ..., T\) is bounded as follows
\[
P_{i,t}^{\min} \leq P_{i,t} \leq P_{i,t}^{\max} (18) \\
\]
where
\[
P_{i,t}^{\min} = \min_{i=1,...,K} P_{i,t,1}^j, P_{i,t}^{\max} = \max_{i=1,...,K} P_{i,t}^j; \forall t = 1, ..., T. \\
\]
It follows that the efficient risk factor over the period \([t, t + 1]\) defined as the relative change in price, can be bounded as follows:

\[
\frac{P_{t+1} - P_{t}}{P_{t}} \leq \frac{P_{t+1} - P_{t+1}}{P_{t+1}} \leq \frac{P_{t+1} - P_{t}}{P_{t}}.
\]  \(19\)

Or alternatively,

\[
R_{i,t+1}^{\min} \leq R_{i,t} \leq R_{i,t+1}^{\max},
\]  \(20\)

where

\[
R_{i,t+1}^{\min} = \frac{P_{t+1} - P_{t+1}}{P_{t+1}}
\]

\[
R_{i,t+1} = \frac{P_{t+1} - P_{t}}{P_{t}}
\]

\[
R_{i,t+1}^{\max} = \frac{P_{t+1} - P_{t}}{P_{t}}.
\]

Hence, only the boundaries of the risk factor can be computed from the observed information. The boundaries are the sole information available to the investor in his decision making. More precisely, he has a range of the return’s possible values for gauging the relative change of the efficient price. In this environment, we suggest representing the risk factor by a closed interval of real numbers. It is thus modeled as a convex compact random set \(\tilde{R}_{i,t+1} = [R_{i,t+1}^{\min}, R_{i,t+1}^{\max}]\), characterized by random bounds. The interval-valued return so defined, models both the stochastic variability of all possible outcomes of the return and the imprecision due to the observation of its efficient value.

3 Model for optimal portfolio selection

As shown in Theorem 2.3, 2.4, 2.5 and 2.6 under Gaussian and \(t\)-Student assumptions of risk factors bounds distributions and for a given risk level \(\theta \in [0, 1]\), the expressions of \(\tilde{V} \tilde{a}R_{1-\theta}\) and \(\tilde{E}S_{1-\theta}\) (generically denoted \(\tilde{R}M_{1-\theta}\) for risk measure at risk level \(\theta\)) are linear combination of the portfolio mean return and volatility. Hence, we rewrite the risk measure \(\tilde{R}M_{1-\theta} = \{\tilde{R}M_{1-\theta}^{l}, \tilde{R}M_{1-\theta}^{u}\}\) as:

\[
\tilde{R}M_{1-\theta}^{l} = \sum_{i=1}^{n} \delta_{i} \mu_{i}^{l} + \psi_{1-\theta}^{l} \sqrt{\sum_{j=1}^{n} \sum_{i=1}^{n} \delta_{j} \sigma_{ij}^{l}}
\]  \(21\)

\[
\tilde{R}M_{1-\theta}^{u} = \sum_{i=1}^{n} \delta_{i} \mu_{i}^{u} + \psi_{1-\theta}^{u} \sqrt{\sum_{j=1}^{n} \sum_{i=1}^{n} \delta_{j} \sigma_{ij}^{u}}
\]  \(22\)

where \(\psi_{1-\theta}^{l}, \psi_{1-\theta}^{u}\) are quantities depending on the risk level and distributions of lower and upper bounds of risk factors, respectively.
In what follows, we denote \( \delta = (\delta_1, \cdots, \delta_n) \in [0, 1]^n \) the vector of portfolio weight. For \( x \in \mathbb{R} \) and \( \mu = (\mu_1, \cdots, \mu_n) \in \mathbb{R}^n \), let \( W(x) \) be the set of \( \delta \) verifying the budget constraint, the short-selling prohibition constraint and ensuring the minimal expected return \( x \). It is formally defined by:

\[
W(x, \mu) = \left\{ \delta = (\delta_n, \cdots, \delta_n) \in [0, 1]^n \mid \sum_{i=1}^{n} \delta_i \mu_i \geq x \text{ and } \sum_{i=1}^{n} \delta_i = 1 \right\}
\]  

(23)

Let us consider the case of an investor interested in the use of these risk measures for portfolio allocation purposes. Our approach thus allows us to discriminate between consumer-protection-oriented schemes, producers-safeguard measures, as well as simultaneous policies.

If this investor is a producer, he will assume that the efficient portfolio risk factor is equal to the lower bounds of the interval-valued one. Indeed, price falls being the problematic outcome for producers, safeguard policies directed towards farmers protection would aim at controlling for descending episodes of price variations.

Since the first element of this \([L, U]\) couple measures the risk induced by the lower bounds, it is used as the risk measure in the portfolio optimization model. It follows that the optimal allocation is the solution of the following producer-relevant portfolio optimization model

\[
\hat{\delta} = \arg \min_{\delta \in W(E_0^l, \mu^l)} \sum_{i=1}^{n} \delta_i \mu_i^l + \psi_{1-\theta} \sqrt{\sum_{j=1}^{n} \sum_{i=1}^{n} \delta_i \delta_j \sigma_{ij}^l}
\]

(24)

where \( \mu^l = (\mu_1^l, \cdots, \mu_n^l) \), \( E_0^l \) the minimal pessimistic expected price variation fixed by the investor.

Equivalently, we now consider the case of a consumer safeguard scheme, which assumes that the efficient value of the portfolio risk factor will coincide with the upper bound of the interval-valued one. Price hikes being the problematic outcome of price evolution for consumers, said scheme should then only address the second element of the \([L, U]\) risk measures couple for the consumers’ portfolio allocation. Hence, the optimal allocation for consumers is solution to the following optimization problem

\[
\hat{\delta} = \arg \min_{\delta \in W(E_0^u, \mu^u)} \sum_{i=1}^{n} \delta_i \mu_i^u + \psi_{1-\theta} \sqrt{\sum_{j=1}^{n} \sum_{i=1}^{n} \delta_i \delta_j \sigma_{ij}^u}
\]

(25)

where \( \mu^u = (\mu_1^u, \cdots, \mu_n^u) \), \( E_0^u \) the minimal pessimistic expected price variation fixed by the investor.

In the case of a public decision-maker concerned with the safeguard of all involved parties, he will be given to simultaneously control for both the lower and upper bounds of the interval-valued risk factors. This new type of agent is thus faced with the task of optimizing the two elements of the \([L, U]\) risk measures couple. In a classical approach, the optimal allocation would
consist in minimizing the sum or the average of the two risk measures on the upper and lower bounds of the risk factors. However, a more consistent solution would be a bi-objective optimization model. By this way, the two elements are optimized separately but simultaneously. Thus, the investor minimizes risks induced by both bounds of the interval-valued factors. In practice, the optimal allocation is solution to the following bi-objective problem presented by

$$
\hat{\delta} = \underset{\delta \in \Lambda}{\text{arg min}} \left( \sum_{i=1}^{n} \delta_{i} \mu_{l i} + \psi_{l - \theta} \left( \sum_{j=1}^{n} \delta_{i} \delta_{j} \sigma_{l ij} \right), \sum_{i=1}^{n} \delta_{i} \mu_{u i} + \psi_{u - \theta} \left( \sum_{j=1}^{n} \delta_{i} \delta_{j} \sigma_{u ij} \right) \right)
$$

where $\Lambda = \mathcal{W}(E_{l 0}^{l}, \mu^{l}) \cap \mathcal{W}(E_{u 0}^{u}, \mu^{u})$, $\mu^{l} = (\mu_{1}^{l}, \ldots, \mu_{n}^{l})$, $\mu^{u} = (\mu_{1}^{u}, \ldots, \mu_{n}^{u})$, $E_{l 0}^{l}$ (resp. $E_{u 0}^{u}$) the minimal producer-relevant (resp. consumer-relevant) expected price variation fixed by the investor.

4 Empirical analysis and results discussion

In this section, we use VaR and ES measures of a portfolio with interval-valued risk factors for allocation purposes, and then proceed to numerical examples. As in the classical case, the investor aims at selecting a combination from a large number of securities, in order to reach an investment goal. The optimal allocation is obtained in practice by minimizing a risk measure for a given minimum expected return.

We carry out an empirical study that illustrates the proposed optimization models. We use real data-sets of monthly prices of 4 agricultural commodities observed in two markets in Niger (a wholesale market and a retail market\footnote{The denominations of “Wholesale” and “Retail” do not imply that purchase can only be made in wholesale or retail sales respectively in each market. In this case, it means that the market is primarily used for wholesale or retail transactions, but identical quantities (large or small) can be purchased in each market.}). This data-sets consist of prices of Millet, Rice, Maize and Sorghum on each market, recorded from January 2007 to May 2013. Figure 1 plots these time series.

The interval-valued risk factors are computed using the method presented in Subsection 2.2. The time series of their bounds are depicted in Figure 2 and their summary statistics presented in Table 1. These summary statistics show that the distributions of lower bounds are characterized by negative mean values and are left-skewed (negative values of skewness). The upper bounds have positive mean values and are right-skewed. Moreover, upper bounds exhibit higher values of variance and kurtosis than the lower bounds.

This is not very surprising because in the case of agricultural commodities, the amplitudes of price swings tend to be higher for price hike episodes than for their decreasing counterparts. Additionally, even if both data-sets exhibit large price increase, the upper bound $[U]$ as computed by our method will always hold the largest increase of the two; hence the greater kurtosis.
Figure 1: Price (expressed in Fcfa/Kg) time series of the considered commodities.

Figure 2: Interval-valued returns of commodities and returns based on mean prices.

Table 1: Statistics summary of lower and upper bounds of interval-valued returns.
We implement the three allocation models presented in Section 4, as well as the classical mean-VaR approach which assumes that the efficient price, unobserved by the investor, is the average value of the various components of the prices data. Variations based on mean prices are used for this latter model and viewed as benchmark for comparison purposes. For each model, minimal mean variations are fixed. Since the summary statistic presented in Table 1 shows kurtosis close to 3 while being inferior, the assumption of normal distribution is adopted for upper and lower bounds of the risk factors. The optimization problems are numerically solved using MATLAB and the obtained results are presented in Table 2.

<table>
<thead>
<tr>
<th>Models</th>
<th>Maize</th>
<th>Millet</th>
<th>Rice</th>
<th>Sorghum</th>
<th>Mean return</th>
</tr>
</thead>
<tbody>
<tr>
<td>Classical model</td>
<td>0.0610</td>
<td>0.0694</td>
<td>0.7625</td>
<td>0.1069</td>
<td>0.005</td>
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<tr>
<td>Producer model</td>
<td>0.1199</td>
<td>0</td>
<td>0.8800</td>
<td>0</td>
<td>-0.1</td>
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<tr>
<td>Consumer model</td>
<td>0.3658</td>
<td>0.0352</td>
<td>0.4536</td>
<td>0.1451</td>
<td>0.2</td>
</tr>
<tr>
<td>Bi-objective model</td>
<td>0.3672</td>
<td>0.1289</td>
<td>0</td>
<td>0.5038</td>
<td>-0.15; 0.2</td>
</tr>
</tbody>
</table>

Table 2: Optimal allocation obtained via the various optimization models.

The classical mean-VaR approach shows the optimal allocation strategy in these commodities. It ensures mitigation of losses due to upwards or downward extreme variations of prices. Results suggest that 76% of available funds be invested in rice, 10% in Sorghum and the remainder equally split between maize and millet (a little over 6% each). The obtained investment setting remains valid and optimal for 0.5% relative price swings.

The producer model selects only two commodities (Maize and Rice) in the optimal allocation whereas the consumer model and the classical one allocate wealth across all considered commodities. The bi-objective model chooses three commodities (Maize, Millet and Sorghum) with 50% of Sorghum. Controlling simultaneously the upper and lower bounds by the use of the bi-objective model, results in the allocation of more than half of available wealth to Sorghum while the use of the producer model does not select this commodity. The other two models allocate a weight ranging between 10% and 15% to this commodity in the optimal allocation situation. This observation can be explained by the fact the interval-valued risk factor of Sorghum exhibits the lowest mean value for the lower bounds and highest mean-value for the upper bounds (see Table 1).

5 Concluding remarks

Portfolio allocation is conventionally carried out using probability theory and by assuming that risk can be described by a unique probability measure. However, under knightian uncertainty, probabilities of events are unknown by the investors and no unique assignment of them can be obtained. This situation is particularly the case in African commodity markets, when an investor is given to make his decision using heterogeneous information. In this situation, a set of probability measures is used in order to reflect the investor’s preferences. For this reason,
we apply an extension of Value-at-Risk and Expected shortfall using a set of probability measures (Mbairadjim Moussa et al., 2014) to evaluate extreme risk of a portfolio of agricultural commodities. In order to address both their stochastic variability and their imprecise nature (induced by the heterogeneity of the used information), our models consider risk factors as interval-valued variables. The resulting risk measures are then implemented for optimal portfolio allocation problems for various types of users.

Two models are proposed; one from a farmers (producer) perspective while the other is from a consuming agent standpoint. Producers aim at limiting investment losses due to price falls. They thus consider that the actual price they will be confronted with will coincide with the lower bound price levels read across the available data. Hence, our results indicated that in order to mitigate losses in such context, they should concentrate investments in the production of rice (at the height of 88% of their capacity), while the remainder is allocated to production of maize. This scheme appears optimal for up to 10% price changes.

Equivalently, consumers aim at mitigation purchasing power losses arising from price hikes. Therefore, they assume that the definite price that they will be faced with on the market is equal to the upper bound of the interval-valued price data. Our computed statistics suggest that, in the described context, consumers ought to invest half of their available income (45%) in the consumption of rice, and then 36% should be allocated to maize, while 15% goes to sorghum and the remainder to millet. This procedure is optimal for up to 20% price swings.

A third model combining the first two is defined as a bi-objective optimization problem, reflecting the purpose of a public decision-maker. Indeed, in the framework of a public food safety policy targeting both types of agents (consumers and producers), 50% of available funds should favor availability of sorghum on the market, while 37% funds maize and the remainder goes to millet. The underlying scheme also implies that given the position that rice holds both in farmers and consumers strategies, the latter commodity need no particular funding or subsidy policy. Standard market functioning mechanisms appear to ensure it’s sustainability and viability for both types of agents. Economic policy efforts are deviated towards Millet and Sorghum markets that appear to be neglected by the individual strategies and then towards the Maize market that holds an intermediate position.

The optimal allocation strategy subsequent to the bi-objective model appears valid for price variations comprised between 15% fall and 20% rise.

Finally, the numerical examples carried out show the effectiveness of our models for practical decision making as compared to the classical procedure.
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Contact:

Stéphane MUSSARD: mussard@lameta.univ-montp1.fr