Are People Risk-Vulnerable?

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Abstract

We report on a within-subject experiment, with substantial monetary incentives, designed to test whether or not people are risk-vulnerable. In the experiment, subjects face the standard portfolio choice problem in which the investor has to allocate part of his wealth between one safe asset and one risky asset. We elicit risk vulnerability by observing each subject’s portfolio choice in two different contexts that only differ by the presence or absence of an actuarially neutral background risk. Our main result is that most of the subjects are risk-vulnerable: 81% chose a less risky portfolio when exposed to background risk. Precisely, 47% invested a strictly smaller amount in the risky asset, while 34% were indifferent. Furthermore, contrasting the predictions provided by competing decision-theoretic models, we conclude that expected utility theory fits better our experimental data.

Key words: Background risk, incomplete markets, portfolio choice, risk vulnerability, lab experiment.

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1 Introduction

Most individuals are exposed to several risks simultaneously. While for some risks individuals can choose their preferred level, there are other risks to which individuals are simply exposed without control, i.e. risks that are non-diversifiable or non-insurable. The fundamental implication of this fact is that there is no risk-free situation for individuals. On the one hand, diversification is limited because of systematic risk. Indeed, economic fluctuations caused by natural disasters, nuclear hazards, financial crisis, wars or popular uprisings, cannot be fully insured. On the other hand, because of informational asymmetries, non-transferability, transaction costs or budget constraints, there exists many idiosyncratic risks for which full insurance is not feasible. In any event, some risks remain inevitably in the background. All such committed but un-resolved risks constitute what is usually called the “background risk”.

Depending on the structure of individuals’ preferences, the presence of background risk may lead to more or less cautious behavior, impacting thereby the price of risk in the economy. Thus, taking into account the background risk to which individuals are exposed can significantly improve our understanding of risk-taking behavior in many economically relevant contexts. Examples include the demand for insurance (Doherty & Schlesinger, 1983; Eeckhoudt & Kimball, 1992), portfolio choices and asset prices (Mehra & Prescott, 1985; Weil, 1992; Finkelshtain & Chalfant, 1993; Franke & al., 1998, 2004; Heaton & Lucas, 1997, 2000), and efficient risk-sharing (Gollier, 1996; Dana & Scarsini, 2007).

The fundamental conjecture upon which this literature rests is that risk-averse individuals consider independent risks as substitutes rather than as complements. According to Gollier & Pratt (1996, p. 1109):

“Conventional wisdom suggests that independent risks are substitutes for each other. In particular, adding a mean-zero background risk to wealth should increase risk aversion to other independent risks. However, risk aversion is not sufficient to guarantee this.”

Relying on von Neumann & Morgenstern (1944)’s expected utility theory (EU), Gollier & Pratt (1996) identified “risk vulnerability” as the weakest restriction to impose on the Bernoulli utility function of a decreasingly risk-averse individual to guarantee that he/she would behave in a more cautious way if an actuarially neutral background risk is added to his/her initial wealth, be it random or not. 1 Since the seminal contribution of Pratt (1964) and Arrow (1971), it is well-known that the absolute risk aversion function governs the risk-taking behavior of individuals with EU preferences. Therefore, the

1 See Gollier & Pratt (1996, Proposition 2, p. 1114; Proposition 4, p. 1120).
comparative-statics properties of risk vulnerability are derived directly from the standard comparative-statics properties of comparative risk aversion.\footnote{See Pratt (1964, Theorem 1, p. 128).}

In the framework of EU, risk vulnerability fits nicely to commonly accepted restrictions that have important and desirable comparative statics properties: risk vulnerability implies decreasing absolute risk aversion (\textit{DARA}) and is implied by both Pratt & Zeckhauser (1987)’s proper risk aversion and Kimball (1993)’s standard risk aversion.\footnote{In fact, \textit{DARA} is equivalent to vulnerability to sure losses, while properness and standardness are equivalent to vulnerability to background risks that reduce expected utility and increase expected marginal utility, respectively.} Since risk vulnerability is necessary to obtain desirable comparative-statics properties in many economic contexts, the question of whether or not most individuals’ behavior actually exhibits risk vulnerability is of paramount interest for economic analysis under EU. But the empirical relevance of the risk vulnerability conjecture is beyond the scope of EU theory as it represents an issue for any decision-theoretic modelling.

In contrast, Quiggin (2003) showed that for the wide class of risk-averse generalized expected utility preferences that exhibit constant risk aversion in the sense of Safra & Segal (1998) and Quiggin & Chambers (1998), independent risks are actually complementary: an individual who is exposed to background risk would be willing to take more foreground risk. In particular, under Yaari (1987)’s dual theory (\textit{DT}), individuals exhibit constant risk aversion and contradict therefore the risk vulnerability conjecture by behaving in a less cautious way when exposed to background risk.

Since alternative theories have different best-guess predictions about the impact of background risk on risk-taking behavior, there is a need for empirical evidence about risk vulnerability in order to contrast predictions with data. More broadly, the validity of the risk vulnerability conjecture is a relevant issue for decision-theoretic modelling in the context of multiple risks.

In this paper we provide experimental evidence about the impact of an actuarially neutral background risk on individuals’ risk-taking behavior. We report on a within-subject experiment, with substantial monetary incentives, designed to test whether or not people are risk-vulnerable. In the experiment, subjects face a simple portfolio choice problem for which they have to allocate part of their wealth between a safe and a risky asset. We elicit risk vulnerability by observing each subject’s portfolio choice in two different contexts that only differ by the presence of an actuarially neutral background risk. Our main result is that most of the subjects are risk-vulnerable: 81\% chose a less risky portfolio when exposed to background risk. Precisely, 47\% invested a strictly smaller amount in the risky asset, while 34\% were indifferent. Thus, only 19\% of the subjects contradict the risk vulnerability conjecture.
In addition, we explore the theoretical predictions about the impact of background risk on the optimal portfolio choice obtained under various preferences representations. We consider, separately, Quiggin (1982)’s rank dependent utility theory (RDU) and its two dual special cases, EU and DT. We show that these theories have contrasted predictions about the impact of background risk on the optimal portfolio choice. We also explore the impact of background risk under Tversky & Kahneman (1992)’s cumulative prospect theory (CPT). We show that, depending on how the reference point of CPT-investors is impacted by the presence of background risk, CPT predicts either risk vulnerability or not. Finally, we consider the generalization of CPT proposed by Schmidt & al. (2008), namely third-generation prospect theory ($PT^3$), in which the reference point can be a lottery. We show that under $PT^3$ the exposition to background risk does not affect the optimal portfolio choice. Hence, contrasting the predictions of competing decision-theoretic models, we conclude that EU theory fits best our experimental data.

The remainder of the paper is organized as follows. Section 2 briefly reviews previous empirical research on risk vulnerability. Section 3 describes our experimental design. Section 4 provides the theoretical foundation for our elicitation procedure of risk vulnerability and presents the predictions for the portfolio choice problem under the above mentioned alternative decision-theoretic models. Our experimental findings are reported in section 5. Section 6 concludes.

2 Evidence about risk vulnerability

To the best of our knowledge, few studies have attempted to question whether or not people are risk-vulnerable. Using naturally occurring data, Guiso & al. (1996) found that investment in risky financial assets responds negatively to income risk, and Guiso & Paiella (2008) showed that individuals who are more likely to face income uncertainty or to become liquidity constrained exhibit a higher degree of absolute risk aversion.

Based on a framed field experiment Harrison & al. (2007) found strong evidence in favor of risk vulnerability for numismatists. They relied on Holt & Laury (2002)’s multiple price list methodology to elicit traders’ risk aversion under three alternative incentives: monetary prizes, graded coins, and ungraded coins which entailed background risk. Their estimates show that using ungraded coins in the lotteries increased sharply the level of risk-aversion of coin traders compared to the conditions where monetary prizes or graded coins were used. They suggest that it would be worth to explore further the extent

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4 A similar field experiment was carried out with students and farmers by Herberich & List (2012).
of their empirical findings on the basis of a controlled laboratory experiment aiming at isolating the impact of background risk on risk-taking behavior.

Lee (2008) reports experimental findings from a laboratory experiment whose aim was to compare the random round payoff mechanism (RRPM) to a system where all rounds are being paid, the accumulated payoff mechanism (APM). In each round subjects had to perform two tasks: task 1 was a risk-taking decision for which subjects had to trade off a higher (lower) probability of winning for a lower (higher) prize. Task 2 was identical except that the event of winning was not determined by a chance event but by the choice made by an opponent player. According to the author the RRPM entails background risk because the subject has to take his decision for task 1 without knowing the outcome of task 2, while in the APM treatment the subject knows his accumulated wealth for task 1 and for task 2. The main finding is that risk-averse subjects tend to behave in a more cautious way under RRPM than under APM. But the data is scarce and the results are not that clear-cut.

Our study is more closely related to Lusk & Coble (2008) who designed explicitly a laboratory experiment to test the risk vulnerability conjecture. Their experiment involved 130 subjects each endowed with $10. The experiment consisted in eliciting subjects’ degree of risk aversion based on Holt & Laury (2002)’s method in a between-subjects design: 50 subjects faced no background risk, 27 subjects faced a zero-mean background risk ($-10, 1/2; 10, 1/2) and 53 subjects faced an unfair background risk ($-10, 1/2; 0, 1/2). The impact of background risk on risk aversion was measured by comparing subjects’ number of safe choices across treatments. The authors found weak evidence of risk vulnerability: the median number of safe choices is identical in the three treatments (6 safe choices) and a slightly greater mean number of safe choices was observed in the zero-mean background risk treatment (5.89 safe choices) and the unfair background risk treatment (5.68 safe choices) compared with the no background risk treatment (5.40 safe choices).  

Finally, a few recent unpublished field-experiments (Cameron & Shah, 2012, Bchir & Willinger, 2013) found empirical evidence in support of the risk vulnerability conjecture.

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Because the unfair background risk they have chosen exhibits non-positive monetary outcomes only, theirs results from the unfair background risk treatment allow a test of DARA rather than a test of risk vulnerability.
3 Experimental design

In order to identify risk vulnerability we rely on a within-subject design. The experiment is based on the standard portfolio choice problem in which the investor has to allocate part of his wealth between a safe asset and a risky asset. The portfolio choice task may be thought as a useful tool to elicit risk-attitudes. It was used for the first time in experiments by Gneezy & Potters (1997), and has been successfully applied to various issues (see, e.g., Haigh & List, 2005; Gneezy & al., 2009; Charness & Gneezy, 2010).

Before performing the portfolio choice task, subjects were first instructed about how their initial cash balance (wealth) would be determined. Since there is striking experimental evidence about the house-money effect on risk-taking (see, e.g., Thaler & Johnson, 1990; Weber & Zuchel, 2001; Martinez & al. 2010) our aim was to control for such a possible effect in our experiment. Although the within-subject method is also intended to wipe out such possible windfall money effects, it is an unanswered question whether such effects may affect risk-taking behavior under background risk. We therefore take the house-money issue seriously. We discuss how we propose to address it below.

Each subject entered the experiment with a controlled level of wealth \( w \). Half of it was credited on a blocked account while the other half was available to the subject for the portfolio choice task. The safe asset had a rate of return equal to 1 and, thus, simply secured the amount invested. On the other hand, the expected return of the risky asset was strictly larger than 1 with a binary random rate of return \( \tilde{k} = (0,1/2; 3,1/2) \) taking either value 0 (losing the amount invested) or value 3 (tripling the amount invested) with equal probability. Letting \( \delta \in [0,1] \) denote the fraction of \( w/2 \) invested in the risky asset, the endogenous random wealth of the investor is given by

\[
\tilde{x} = w + \delta \left[ \tilde{k} - 1 \right] \frac{w}{2}
\]  

Each subject faced the portfolio choice task twice in two different situations, labeled \( A \) and \( B \), which were presented sequentially. It was made clear that only one of the two situations would apply for real at the end of the experiment, the choice of the relevant situation being decided on a random basis. Situation \( A \) involved no background risk while, in situation \( B \), each subject was exposed to an independent, additive and actuarially neutral background risk \( \tilde{y} = (-y,1/2; y,1/2) \) impacting his blocked account. We chose the level of background risk in such a way that subjects could eventually lose their whole wealth in the blocked account, i.e. \( y = w/2 \). The balance of a subject’s blocked account was certain in situation \( A \) (equal to \( w/2 \)) and risky in situation \( B \) (either equal to 0 or \( w \) with equal probability). Thus, because the two situ-
ations $A$ and $B$ only differed by the presence or absence of background risk, our experimental design allows us to elicit each subject’s risk vulnerability in an unambiguous way by comparing his/her investment in situation $A$ and in situation $B$. We control for a possible order effect by randomizing the sequence of situations: approximately half of the subjects faced situation $A$ first, while the other half faced situation $B$ first. Our main result is unaffected by the ordering of the two situations.

Because we relied on the RRPM procedure for selecting the relevant situation at the end of the experiment, one could actually argue that we implemented a second (uncontrolled) background risk, as in Lee (2008)’s experiment. While this might be the case, it does not affect our conclusion. Indeed, whether or not an uncontrolled background risk was present in our experiment, it is wiped out by our within-subject design, because it does not differ between situation $A$ and situation $B$. The background risk generated by the RRPM procedure merely cumulates with each subject’s own background risk that he/she brings with him/her to the lab and for which we are unable to control for. With the exception of the order effect, the impact of background risk is therefore fully captured, all other things equal, by our experimental design.

279 student-subjects participated in our experiments. Participants were selected randomly from a large pool of over 3000 volunteers. Real monetary incentives were offered to all participants. To allow for robustness check we report on two experiments, labeled $Exp.1$ and $Exp.2$, where we deliberately varied many aspects of the experimental design. $Exp.1$ was run as a paper and pencil session involving 91 subjects while $Exp.2$ was a computerized experiment involving 188 subjects. In $Exp.2$, preliminary to the portfolio choice task, each subject had to work in order to accumulate wealth ($w = 20$) by performing a tedious task. In contrast, in $Exp.1$ subjects’ wealth was a windfall endowment ($w = 100$) provided by the experimenter. The reason for including this first stage in $Exp.2$ was to control for the house-money effect that could have affected the results of $Exp.1$. Finally, in $Exp.1$, which involved high stakes, only 10% of the participants were randomly selected to be paid out

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6 The work consisted in reporting the number of times number “1” appeared in a matrix containing strings of “0’s” and “1’s”. Ten different matrixes with varying sizes had to be counted this way during a limited period of time. At the end of this preliminary stage, only subjects who had completed the task correctly for all ten matrixes received the flat reward of €20. Those who failed were instructed that they could stay in the experiment until the end, but that they would play with fictitious money (9% of the subjects).

7 In the experiments the amount invested in the risky asset was restricted to integer values. Therefore the choice-space differed somehow between $Exp.1$ and $Exp.2$. With $w/2 = 50$ in $Exp.1$ and $w/2 = 10$ in $Exp.2$, the corresponding choice-set in $Exp.1$ was $\{0, 0.02, ..., 1\}$ and $\{0, 0.1, ..., 1\}$ in $Exp.2$. That is, subjects could invest fractions of 2% in $Exp.1$ but only fractions of 10% in $Exp.2$. 

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for real. By contrast, in Exp.2, where stakes were much lower, all participants were paid according to their earnings. As we show in the result section, despite these strong design differences, the results of the two experiments almost perfectly match.

4 Theoretical predictions

In this section we derive theoretical predictions about risk vulnerability. The issue of risk vulnerability has been extensively discussed within the framework of EU, but very sparse outside. It is therefore of interest to contrast the predictions of EU to the predictions of alternative theories: DT, RDU, CPT and PT. As one can expect, EU and DT lead to sharp but opposite predictions. On the other hand, more general forms of RDU provide less clean predictions. Predicting the investor’s behavior under CPT is more problematic, since it depends on the definition of the reference point under background risk. Hence, we consider the predictions of the most recent version of CPT, PT, which allows the reference point to be random. Under PT, we show that the investor is indifferent with respect to the introduction of the background risk.

4.1 Portfolio choice and risk vulnerability

Let us consider an investor seeking to maximize a general preferences function \( v \) defined over random wealth. Without background risk (situation A) the investor’s optimal portfolio is given by

\[
\delta^A = \arg \max_{\delta \in [0,1]} v(\bar{x})
\]  
(2)

Now suppose that the investor is exposed to the actuarially neutral background risk \( \bar{y} \) (situation B). His/her random wealth is now \( \bar{x} + \bar{y} \), and his/her optimal portfolio is given by

\[
\delta^B = \arg \max_{\delta \in [0,1]} v(\bar{x} + \bar{y})
\]  
(3)

In this framework, risk vulnerability amounts to the comparison of the two optimal levels of investment \( \delta^A \) and \( \delta^B \). We rely on two categorizations: a coarse categorization which distinguishes between risk-vulnerable and non-risk-vulnerable investors, and a fine categorization that adds a further distinction within the risk-vulnerable category between investors that are strictly-risk-vulnerable and those that are considered as indifferent. Definition 1 below summarizes the possible types of behavior.
Definition 1 The behavior of the investor is risk-vulnerable if $\delta^A \geq \delta^B$, strictly-risk-vulnerable if $\delta^A > \delta^B$, indifferent if $\delta^A = \delta^B$ and non-risk-vulnerable if $\delta^A < \delta^B$.

4.2 Expected utility theory

Under $EU$, the preferences function is linear in the probabilities and takes the following form:

$$v(\bar{x}) = Eu(\bar{x})$$

(4)

where $u$ is a strictly increasing real-valued Bernoulli utility function defined over final wealth. Following previous literature (Kihlstrom et al., 1981; Nachman, 1982; Pratt, 1988; Pratt & Zeckhauser, 1987; Gollier & Pratt, 1996) it is convenient under background risk to define an indirect Bernoulli utility function $U(x) = Eu(x + \bar{y})$. Thus, when the $EU$-investor is exposed to the background risk, his/her preferences function becomes:

$$v(\bar{x} + \bar{y}) = EU(\bar{x})$$

(5)

The Kuhn-Tucker first-order conditions for situation $A$ are:

$$\frac{u'(x_1)}{u'(x_2)} \begin{cases} \geq 2 & \text{if } \delta^A < 1 \\ \leq 2 & \text{if } \delta^A > 0 \end{cases}$$

(6)

where $x_1 = [2 - \delta] w/2$ and $x_2 = [1 + \delta] w$ represent the investor’s wealth in the bad state of the world (unsuccessful investment) and in the good state of the world (successful investment), respectively. The optimal portfolio for situation $A$ can then be summarized as follows:

$$\delta^A \begin{cases} = 1 & \text{if } \frac{u'(x_1)}{u'(x_2)} < 2 \\ \in [0,1] & \text{if } \frac{u'(x_1)}{u'(x_2)} = 2 \\ = 0 & \text{if } \frac{u'(x_1)}{u'(x_2)} > 2 \end{cases}$$

(7)

First note that, because the excess rate of return is strictly positive with $E\tilde{r} > 1$, a risk-neutral individual would always choose the maximum possible level of

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8 The rational for this formulation is that examining the effect of the introduction of background risk is equivalent to examining differences between preferences represented by $u$ and $U$. An investor exposed to the background risk and having preferences represented by $u$ would act as a non-exposed investor with preferences represented by $U$.

9 Substituting $U$ for $u$ and $\delta^B$ for $\delta^A$ in (6) and (7) gives the analogous conditions for situation $B$. 

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investment. On the other hand, because \( \delta = 0 \iff x_1 = x_2 \), it is apparent from (7) that, under monotonic preferences, a zero-investment cannot be an optimal choice for an EU-investor.\(^{10}\) Observe also that, because \( \delta > 0 \iff x_1 < x_2 \), the ratio of marginal utilities in (7) is greater than one under risk aversion and is increasing with the investor’s degree of risk aversion. Hence, recognizing that \( U \) is at least as risk-averse than \( u \) if and only if\(^{11}\)

\[
\frac{u'(x_1)}{u'(x_2)} \leq \frac{U''(x_1)}{U''(x_2)} \quad \forall x_1 \leq x_2
\]

(8)

we get that the investor is risk-vulnerable if and only if the ratio of marginal utilities is larger in the presence of background risk. According to (7), this suggests that \( \delta^A \geq \delta^B \). Gollier & Pratt (1996, Definition 1, p.1112) equivalently defined risk vulnerability by the following inequality:

\[
r(x) = -\frac{u''(x)}{u'(x)} \leq -\frac{U''(x)}{U'(x)} = R(x) \quad \forall x
\]

(9)

Thus, the Arrow-Pratt framework of comparative risk aversion fully applies as if \( u \) and \( U \) corresponded to the preferences of two different investors. In particular the following well-known result applies.\(^{12}\)

**Proposition 2** Under EU the following statements are equivalent:

- \( U \) is at least as risk-averse than \( u \) (\( r \leq R \)).
- The behavior of the investor is risk-vulnerable (\( \delta^A \geq \delta^B \)).

As mentioned by Gollier & Pratt (1996), all commonly used Bernoulli utility functions that satisfy non-increasing harmonic absolute risk aversion (HARA) exhibit risk vulnerability. Assuming constant absolute risk aversion (CARA), the additive background risk \( \bar{y} \) has obviously no impact on absolute risk aversion. Thus, CARA-investors would be indifferent to the background risk. On the other hand, DARA-investors are risk-vulnerable if:

\[
\frac{r(x) - r(x+y)}{r(x+y) - r(x)} \leq \frac{u'(x-y)}{u'(x+y)} \quad \forall x, y > 0
\]

(10)

The r.h.s. of inequality (10) corresponds to the investor’s marginal rate of substitution between his/her wealth in the case of a bad outcome of the background risk and his/her wealth in the case of a good outcome of the background risk.

\(^{10}\)In our experiments, we observed that 8% (16%) of the subjects chose a zero-investment in situation \( A \) (\( B \)). On the other hand, 17% (11%) chose a maximum-investment in situation \( A \) (\( B \)).

\(^{11}\)See Pratt (1988, Equation 2, p. 398).

\(^{12}\)See Pratt (1964, Theorem 1, p. 128) and Gollier & Pratt (1996, Proposition 1, p. 1112).
risk. Under monotonic preferences and risk aversion, this ratio is obviously greater than one. Therefore, a sufficient condition for a DARA-investor to be risk-vulnerable is that the ratio of changes in absolute risk aversion in the l.h.s. of inequality (10) is smaller than one. Because the numerator, \( r(x) - r(x+y) \), is the increase in absolute risk aversion due to the loss \(-y\) at wealth level \(x+y\) while the denominator, \( r(x-y) - r(x) \), is the increase in absolute risk aversion due to the loss \(-y\) at wealth level \(x\), it follows that the convexity of absolute risk aversion is sufficient for (10) to hold. Therefore, as observed by Gollier & Pratt (1996), decreasing and convex absolute risk aversion constitutes a simple and intuitive sufficient condition for risk vulnerability in the EU framework.\(^{13}\)

Let us illustrate by considering the widely used power utility function exhibiting constant relative risk aversion (CRA) with parameter \(\gamma\):

\[
    u(x) = \begin{cases} 
    x^{1-\gamma} & \text{if } \gamma > 0 \\
    \ln x & \text{if } \gamma = 1 
    \end{cases} \quad \forall x > 0 
\]

The predicted impact of background risk on the portfolio choice is illustrated as a function of the CRA parameter in Figure 1.\(^{14}\) As the CRA parameter increases, the optimal investment curve is first horizontal and then non-increasing. Because the CRA Bernoulli utility function exhibits risk vulnerability, the investor behaves in a more cautious way after the introduction of the background risk and the optimal investment curve shifts down.

\(^{13}\)In particular, decreasing and convex risk aversion means that the level of investment in the risky asset is increasing and concave in wealth.

\(^{14}\)In all our numerical illustrations, we used the parameters of Exp.2 where \(w = \mathcal{E}20\) and \(y = \mathcal{E}10\), with \(\delta \in \{0,0.1,...,1\}\).
4.3 Dual theory

Assume now that the investor behaves according to $DT$. As opposed to $EU$ theory, under $DT$ the preferences function is linear in monetary outcomes and non-linear in probabilities. By convention, but with no loss of generality, outcomes are ordered from the smallest to the largest. Without background risk, only two outcomes are possible, and we have: $x_1 = x_2 \iff \delta = 0$ and $x_1 < x_2 \iff \delta > 0$. The probability weight attributed to wealth $x_i$ is then determined as follows:

$$
\pi_i = g(\Pr(\bar{x} \leq x_i)) - g(\Pr(\bar{x} < x_i)) \quad \forall i = 1, 2 \tag{12}
$$

where $g : [0, 1] \to [0, 1]$ is a strictly increasing probability weighting function satisfying $g(0) = 0$ and $g(1) = 1$. Thus, we have $\pi_1 = 1 - \pi_2 = g(1/2)$. Recognizing that the probability weights are positive and sum to one, Yaari (1987) interpreted the preferences function as a “corrected mean” of $\bar{x}$:

$$
v(\bar{x}) = \sum_{i=1}^{2} \pi_i x_i \tag{13}
$$

If the probability weighting function is concave (convex), the $DT$-investor behaves pessimistically (optimistically), as though an unsuccessful investment
is more (less) likely than it really is: \( \pi_1 > 1/2 > \pi_2 \) (\( \pi_1 < 1/2 < \pi_2 \)).\(^{15}\) Furthermore, because the preferences function in (13) is linear in the level of investment, \( DT \) typically leads to corner solutions and the optimal portfolio is fully determined by the probability weight of an unsuccessful investment:

\[
\delta^A = \begin{cases} 
1 & \text{if } \pi_1 < \frac{2}{3} \\
\in [0, 1] & \pi_1 = \frac{2}{3} \\
0 & \pi_1 > \frac{2}{3}
\end{cases} \tag{14}
\]

In the words of Yaari (1987), \( DT \) predicts “plunging”, rather than diversification. \( DT \)-investors stay put until plunging becomes justified.\(^{16}\) From (14) we see that \( DT \)-investors who are not too pessimistic with \( \pi_1 < 2/3 \) would choose the maximum possible level of investment in the risky asset. On the other hand, \( DT \)-investors who are strongly pessimistic with \( \pi_1 > 2/3 \) would choose to invest zero in the risky asset. Thus, it is only in the very special case where \( \pi_1 = 2/3 \) that \( DT \)-investors are indifferent between all possible levels of investment and, hence, would choose a non-extreme level of investment.

In the presence of background risk, there are four (equiprobable) possible outcomes: \( x_{11} = x_1 - y, x_{12} = x_1 + y, x_{21} = x_2 - y \) and \( x_{22} = x_2 + y \), where \( y = w/2 \). The weight attributed to wealth \( x_{ij} \) is written:

\[
\pi_{ij} = g (\Pr (\bar{x} + \bar{y} \leq x_{ij})) - g (\Pr (\bar{x} + \bar{y} < x_{ij})) \quad \forall i, j = 1, 2 \tag{15}
\]

with \( \sum_{i=1}^{2} \sum_{j=1}^{2} \pi_{ij} = 1 \). The preferences function is then defined as follows:

\[
v (\bar{x} + \bar{y}) = \sum_{i=1}^{2} \sum_{j=1}^{2} \pi_{ij} x_{ij} \tag{16}
\]

As without background risk, the optimal portfolio is fully determined by the

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\(^{15}\) As pointed out by Yaari (1987, p. 108), however, “this is not a case where probabilities are being distorted in the agent’s perception ... this essay deals with how perceived risk is processed into choice, and not with how actual risk is processed into perceived risk.”

\(^{16}\) Plunging must not be confused with risk-seeking, which means never stay put under \( EU \).
probability weight of an unsuccessful investment: \[\delta^B = \begin{cases} 
1 & \in [0, 1] \\
0 & \text{if } \sum_{j=1}^{2} \pi_{1j} < \frac{2}{3} \\
1 & \text{if } \sum_{j=1}^{2} \pi_{1j} > \frac{2}{3} 
\end{cases} \tag{17}\]

From the comparison of the optimal portfolio choices in (14) and (17), it appears that $DT$-investors may be risk-vulnerable only if the background risk makes them more pessimistic regarding the outcome of their investment, that is, if $\sum_{j=1}^{2} \pi_{1j} \geq \pi_1$. However, as shown by Safra & Segal (2008), in general, individuals who behave according to $DT$ cannot be risk-vulnerable. \[\text{In the present context, it can be shown that the introduction of the background risk cannot make } DT\text{-investors more pessimistic whenever the probability weight-}
\text{ing function is concave.}\]

Observe also that the concavity of the probability weighting function is a necessary and sufficient condition for strong risk aversion (see, e.g., Chew & al., 1987). Therefore, under strong risk aversion, $DT$ cannot predict strictly-risk-vulnerable behavior. We provide a numerical example to illustrate this result. To generate probability weights, we use Safra and Segal (2008)’s single parameter functional form:

$g(p) = p^\alpha \quad \forall p \in [0, 1] \tag{18}$

where $\alpha > 0$. This function is concave (convex) for $\alpha < 1$, so that relatively more (less) weight is given to probabilities associated with bad outcomes. Thus, if $\alpha < 1$ ($\alpha > 1$), the individual is pessimistic (optimistic). The predicted impact of background risk on the portfolio choice is illustrated as a function of the parameter $\alpha$ in Figure 2.

\[\text{Things are slightly more complicated in the presence of background risk because the ranking of outcomes depends on the level of investment and is therefore endogenous. Whatever the level of investment, } x_{11} \text{ and } x_{22} \text{ represent the lowest payment and the highest payment, respectively. Thus, } \pi_{11} = g(1/4) \text{ and } \pi_{22} = 1 - g(3/4). \text{ On the other hand, the ordering of } x_{12} \text{ and } x_{21} \text{ depends on } \delta: x_{12} < (>) x_{21} \iff \delta > (\leq) 2/3. \text{ Thus, } \pi_{12} = g(1/2) - g(1/4) \text{ and } \pi_{21} = g(3/4) - g(1/2) \text{ if } \delta > 2/3, \text{ while } \pi_{12} = g(3/4) - g(1/2) \text{ and } \pi_{21} = g(1/2) - g(1/4) \text{ if } \delta < 2/3. \]

\[\text{See Safra & Segal (2008, Proposition 2, p. 1152). The result is based on a property called “stochastic B3” which is similar to risk vulnerability.}\]

\[\text{If } \delta > 2/3 \text{ then } \sum_{j=1}^{2} \pi_{1j} = \pi_1 = g(1/2). \text{ On the other hand, if } \delta < 2/3 \text{ then } \sum_{j=1}^{2} \pi_{1j} \geq \pi_1 \text{ is equivalent to } [g(1/4) + g(3/4)]/2 \geq g(1/2). \text{ By Jensen’s inequality, this cannot be true whenever } g \text{ is concave.}\]
4.4 Rank-dependent utility

EU and DT are special cases of RDU. Thus, under RDU the preferences function is non-linear in both monetary outcomes (as under EU) and probabilities (as under DT) and takes the following general form:

$$v(\bar{x}) = \sum_{i=1}^{2} \pi_i u(x_i)$$

To illustrate, we use the CRRA Bernoulli utility function (11) together with the probability weighting function (18). The RDU-investor’s preferences are therefore defined by two parameters: the degree of risk aversion captured by the parameter $\gamma$ (the curvature of the Bernoulli utility function $u$) and the degree of pessimism captured by the parameter $\alpha$ (the curvature of the probability weighting function $g$). Keeping constant the CRRA parameter at $\gamma = 0.75$, the predicted impact of background risk on the portfolio choice is illustrated as a function of the parameter $\alpha$ in Figure 3. First note that the EU-side of the RDU-investor is risk-vulnerable because the Bernoulli utility function (11) exhibits decreasing and convex absolute risk aversion. On the other hand, for $\alpha < 1$ the probability weighting function (18) is concave and the DT-side of the RDU-investor cannot be risk-vulnerable. Thus, whenever $\alpha < 1$, the two sides of the RDU-investor act against each other and the RDU-investor may be risk-vulnerable or not. However, as $\alpha$ increases and approaches 1, the DT-side becomes less influent. It is indeed apparent from
Figure 3 that the \textit{RDU}-investor is first non-risk-vulnerable and moves towards risk vulnerability as $\alpha$ increases. For $\alpha > 1$, the probability weighting function (18) is convex and the \textit{DT}-side of the \textit{RDU}-investor is risk-vulnerable. Thus the two sides of the \textit{RDU}-investor reinforce each other and the \textit{RDU}-investor becomes unambiguously risk-vulnerable.

\textbf{Figure 3. Predicted portfolio choice (RDU)}

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure3.png}
\caption{Predicted portfolio choice (RDU)}
\end{figure}

\textit{A} = without background risk; \textit{B} = with background risk

\subsection{4.5 Cumulative prospect theory}

\textit{CPT} belongs to the class of rank-dependent models. As under \textit{RDU}, the preferences function is non-linear in both monetary outcomes and probabilities, but since asset integration does not apply under \textit{CPT}, the carriers of value are the variations of wealth with respect to a reference point $x^*$. This is captured by the value function introduced by Kahneman & Tversky (1979) that is defined over gains and losses (above and below the reference point):

\begin{equation}
\begin{aligned}
u (x) =
\begin{cases}
u^+ (x - x^*) & \text{if } x \geq x^* \\
-\nu^- (x^* - x) & \text{if } x \leq x^*
\end{cases}
\end{aligned}
\end{equation}

where $u (x^*) = u^- (0) = u^+ (0) = 0$. The value function is increasing, concave for gains and convex for losses. Without background risk the reference point is clearly the level of initial wealth, i.e. $x^* = w$, which is also the sure level of wealth that can be obtained if the individual stays put by choosing to invest zero in the risky asset. In this context, we have: $x_1 = x^* = x_2 \iff \delta = 0$ and
Thus, according to the realized state of the rate of return of the risky asset, the investor records, with equal probability, either a loss or a gain:

\[ x^* - x_1 = \frac{1}{2} \delta w \] (loss) \hspace{1cm} (21)

or

\[ x_2 - x^* = \delta w \] (gain) \hspace{1cm} (22)

A second feature of CPT is that the probability weighting function differs for gains and for losses. The probability weighting function is defined over the cumulative distribution function for losses

\[ \pi^-_1 = g^- (\Pr (\bar{x} \leq x_1)) - g^- (\Pr (\bar{x} < x_1)) \] \hspace{1cm} (23)

and over the decumulative distribution function for gains

\[ \pi^+_2 = g^+ (\Pr (\bar{x} \geq x_2)) - g^+ (\Pr (\bar{x} > x_2)) \] \hspace{1cm} (24)

where \( g^- \) and \( g^+ \) are strictly increasing functions from the unit interval into itself satisfying \( g^- (0) = g^+ (0) = 0 \) and \( g^- (1) = g^+ (1) = 1 \). Thus, we have \( \pi^-_1 = g^- (1/2) \) and \( \pi^+_2 = g^+ (1/2) \). The preferences function takes therefore the following form:

\[ v (\bar{x}) = g^+ (1/2) u^+ (x_2 - x^*) - g^- (1/2) u^- (x^* - x_1) \] \hspace{1cm} (25)

When exposed to the background risk, it is unclear what the CPT-investor’s reference point might be. If we rely on the same benchmark than without background risk, i.e. the level of wealth that can be reached if the investor chooses to stay put, there are now two possible candidates, denoted \( \bar{x}^* = x^* - y \) and \( \bar{x}^* = x^* + y \), which are respectively below and above the reference point without background risk: \( \bar{x}^* < x^* < \bar{x}^* \). These two possible reference points may be thought as pessimistic and optimistic, respectively. Indeed, if the CPT-investor takes into account \( \bar{x}^* (\bar{x}^*) \) as the reference point, he evaluates the outcomes of his investment decision as if he believed that the bad (good) outcome of the background risk will be realized for sure.

Hence, intuition suggests that a pessimistic (optimistic) CPT-investor would be risk-vulnerable (non-risk-vulnerable). The reasoning is the following: if the individual is pessimistic (optimistic) about the outcome of the background risk, most outcomes will be above (below) his/her reference point \( \bar{x}^* (\bar{x}^*) \). Because the value function is concave (convex) in the domain of gains (losses), pessimistic (optimistic) CPT-investors will be more (less) reluctant to invest in the risky asset when exposed to the background risk. However, one needs to take also into account the property that the probability weighting function differs in the loss and the gain domains. Table 1 provides numerical illustrations
for which this intuition is verified. Probability weights have been generated using Prelec (1998)’s single parameter function

\[ g(p) = \exp \{ - [- \ln(p)]^\theta \} \quad \forall p \in [0, 1] \tag{26} \]

with \( \theta^+ = 0.5 \) for gains and \( \theta^- = 0.65 \) for losses. Moreover, we used power value functions \( u^+(x) = x^\alpha \) for gains and \( u^-(x) = \lambda x^\beta \) for losses, with three standard cases for parameters \( \alpha, \beta \) and \( \lambda \). Without background risk, \( CPT \) predicts a positive but very small investment in the risky asset for the three sets of parameter values. In the presence of background risk, with the pessimistic reference point \( x^* \), the optimal investment is even lower and the investor’s behavior exhibits risk vulnerability. On the contrary, with the optimistic reference point \( \tilde{x}^* \), the maximum investment becomes optimal. Thus, depending on the reference point chosen in the presence of background risk, \( CPT \) predicts either strictly-risk-vulnerable or non-risk-vulnerable behavior.

### Table 1. Predicted Portfolio Choice (\( CPT \))

<table>
<thead>
<tr>
<th>Reference point</th>
<th>Type of behavior</th>
<th>( (\delta^A) )</th>
<th>( (\delta^B) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stay put &amp; Pessimistic</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( x^* )</td>
<td>Strictly-RV</td>
<td>( \delta^A \geq \delta^B )</td>
<td>0.005</td>
</tr>
<tr>
<td>Stay put &amp; Optimistic</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \tilde{x}^* )</td>
<td>Non-RV</td>
<td>( \delta^A &lt; \delta^B )</td>
<td>0.009</td>
</tr>
<tr>
<td>Stay put</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \tilde{x}^* )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \bar{x}^* )</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

RV = risk-vulnerable; Ind = indifferent; A = without background risk; B = with background risk

An alternative way of choosing the reference point in the presence of background risk is to rely on Schmidt & al. (2008)’s \( PT^3 \), the third generation of Kahneman & Tversky (1979)’s Prospect Theory, which allows the reference point to be random. Recalling that there are four equiprobable possible outcomes in the presence of background risk, the random reference point can be represented by the following lottery: \( \tilde{x}^* = (x_{11}^*, 1/4; x_{12}^*, 1/4; x_{21}^*, 1/4; x_{22}^*, 1/4) \).

In order to determine the reference point in each state of the world, one needs to define a “reference act”. As without background risk, the natural reference act in our setting is to stay put, that is to choose a zero-investment. Thus, if \( \delta = 0 \), then \( x_{11}^* = x_{21}^* = x^* \) in the case of a bad outcome of the background.
risk and \( x_{12}^* = x_{22}^* = \bar{x}^* \) in the case of a good outcome of the background risk. Therefore, the reference act leads to the same two equiprobable outcomes as without background risk:

\[
x_{11}^* - x_{11} = x_{12}^* - x_{12} = x^* - x_1 \quad \text{(loss)} \quad (27)
\]
or

\[
x_{21} - x_{21}^* = x_{22} - x_{22}^* = x_2 - x^* \quad \text{(gain)} \quad (28)
\]

Thus, the impact of the background risk is fully absorbed by the randomness of the reference point, and the preferences function takes exactly the same form than without background risk: \( v(\bar{x} + \bar{y}) = v(\bar{x}) \). As a result, \( PT^J \)-investors are indifferent to the presence of background risk.

5 Results

According to definition 1, we rely on two levels of categorization of our subjects: a coarse categorization and a fine categorization. The coarse categorization adopts Gollier & Pratt (1996)'s original definition by separating risk-vulnerable subjects for whom independent risks are substitutes (or independent), i.e. \( \delta^A \geq \delta^B \), from non-risk-vulnerable subjects for whom they are complements, i.e. \( \delta^A < \delta^B \). The fine categorization divides the risk-vulnerable category further into strictly-risk-vulnerable subjects for whom independent risks are substitutes, i.e. \( \delta^A > \delta^B \), and indifferent subjects for whom they are independent, i.e. \( \delta^A = \delta^B \).

Among the 279 subjects who have participated in our experiments, 11% chose the same limit-investment in both situation \( A \) and situation \( B \). Precisely, 4% chose \( \delta^A = \delta^B = 0 \) and 7% chose \( \delta^A = \delta^B = 1 \). Because the range of possible investments was bounded in the experiment, these subjects may either be categorized as indifferent, strictly-risk-vulnerable or non-risk-vulnerable. Since these subjects are likely to be of any of these types, we decided to categorize them as indifferent.\(^{20}\)

Result 1. A large majority of subjects are risk-vulnerable, i.e. behave as if independent risks were substitutes (or independent). Risk vulnerability is robust both to treatment effects and to experimental manipulations.

Support for result 1

\(^{20}\)If we exclude the 30 subjects who have chosen the same limit-investment in the two situations, 78% of the remaining 249 subjects are risk-vulnerable, 53% are strictly-risk-vulnerable, 26% are indifferent and 22% are non-risk-vulnerable.
Table 2 summarizes the results for the pooled data of the two experiments and treatments. We find that a non-trivial portion of the subjects invested an equal or lower amount in the risky asset when exposed to background risk. According to the coarse categorization, 81% of the subjects are risk-vulnerable, while 19% are non-risk-vulnerable. According to the fine categorization, 47% are strictly-risk-vulnerable, while 34% are indifferent. Our main result is that the relative frequency of risk-vulnerable subjects is significantly larger than 50% in both experiments and for both treatments (binomial test, \( p = 0.000 \)).

<table>
<thead>
<tr>
<th>Type of behavior</th>
<th>RV</th>
<th>Strictly-RV</th>
<th>Ind</th>
<th>Non-RV</th>
<th>All</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \delta^A \geq \delta^B )</td>
<td>81%</td>
<td>47%</td>
<td>34%</td>
<td>19%</td>
<td>100%</td>
</tr>
<tr>
<td>( \delta^A &gt; \delta^B )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \delta^A = \delta^B )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \delta^A &lt; \delta^B )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Frequencies

Mean \( \delta^A \)

Mean \( \delta^B \)

Nb. of obs.

RV = risk-vulnerable; Ind = indifferent; A = without background risk; B = with background risk

Figure 4 provides further evidence in support for result 1. The cumulative distribution function of investment without background (CDF of \( \delta^A \)) first-order stochastically dominates the CDF of \( \delta^B \). Because \( \Pr(\delta^A \leq \delta) < \Pr(\delta^B \leq \delta) \) for any \( \delta < 1 \), the dominance relation is strong. In words, for each possible level of investment \( \delta \), the probability that a randomly selected subject chose to invest less than \( \delta \) is always larger with background risk than without.
Figure 4. CDF of observed portfolio choices (pooled data)

Table 3 presents the data separately for each treatment. We observe that whether subjects are first exposed to situation A (without background risk – treatment $AB$) or first exposed to situation B (with background – treatment $BA$) is irrelevant. Because there is no order effect with respect to the frequency of risk-vulnerable subjects (Fisher exact test, 5%) we can pool the data of the two treatments.
Table 3. Observed portfolio choices (across treatments)

<table>
<thead>
<tr>
<th>Type of behavior</th>
<th>RV</th>
<th>Strictly-RV</th>
<th>Ind</th>
<th>Non-RV</th>
<th>All</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\delta^A \geq \delta^B$</td>
<td>$\delta^A &gt; \delta^B$</td>
<td>$\delta^A = \delta^B$</td>
<td>$\delta^A &lt; \delta^B$</td>
<td></td>
</tr>
<tr>
<td>Treatment AB</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Frequencies</td>
<td>80%</td>
<td>44%</td>
<td>36%</td>
<td>20%</td>
<td>100%</td>
</tr>
<tr>
<td>Mean $\delta^A$</td>
<td>0.492</td>
<td>0.522</td>
<td>0.455</td>
<td>0.350</td>
<td>0.464</td>
</tr>
<tr>
<td>Mean $\delta^B$</td>
<td>0.327</td>
<td>0.223</td>
<td>0.455</td>
<td>0.601</td>
<td>0.381</td>
</tr>
<tr>
<td>Nb. of obs.</td>
<td>107</td>
<td>59</td>
<td>48</td>
<td>26</td>
<td>133</td>
</tr>
<tr>
<td>Treatment BA</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Frequencies</td>
<td>81%</td>
<td>49%</td>
<td>32%</td>
<td>19%</td>
<td>100%</td>
</tr>
<tr>
<td>Mean $\delta^A$</td>
<td>0.637</td>
<td>0.679</td>
<td>0.573</td>
<td>0.304</td>
<td>0.574</td>
</tr>
<tr>
<td>Mean $\delta^B$</td>
<td>0.420</td>
<td>0.322</td>
<td>0.573</td>
<td>0.614</td>
<td>0.457</td>
</tr>
<tr>
<td>Nb. of obs.</td>
<td>118</td>
<td>72</td>
<td>46</td>
<td>28</td>
<td>146</td>
</tr>
</tbody>
</table>

RV = risk-vulnerable; Ind = indifferent; A = without background risk; B = with background risk

In order to highlight the impact of our experimental manipulations, Table 4 summarizes the data of Exp.1 and Exp.2 separately. Despite the many differences in design features, the two experiments produce an equal frequency of risk-vulnerable subjects (Fisher exact test, 5%). The null hypothesis of equal distributions of percentages invested in Exp.1 and Exp.2 cannot be rejected neither for situation A nor for situation B (KS test, two-sided, 5%). Furthermore, the null hypothesis of equal distributions for $\delta^A - \delta^B$ in Exp.1 and Exp.2 cannot be rejected either (KS test, two-sided, 5%). We therefore conclude that Exp.1 and Exp.2 produce almost exactly the same results.\(^{21}\)

\(^{21}\)Note, however, that the finer categorization of risk-vulnerable subjects into strictly-risk-vulnerable and indifferent types reveals a slight difference between Exp.1 and Exp.2. As can be seen from Table 4, the percentage of strictly-risk-vulnerable subjects drops from 52% in Exp.1 to 45% in Exp.2 while at the same time the percentage of indifferent subjects increases from 29% in Exp.1 to 36% in Exp.2. A plausible reason for such a difference might be the finer scale of the choice-space in Exp.1, where $\delta \in \{0, 0.02, \ldots, 1\}$, compared to Exp.2, where $\delta \in \{0, 0.1, \ldots, 1\}$. The minimum reduction possibility was 0.02 in Exp.1 whereas it was 0.10 in Exp.2. Thus, in Exp.2 some subjects might have felt a sharper constraint for adjusting their level of investment in the risky asset. For instance in Exp.2, subjects who might have been willing to reduce their investment by 0.05 in situation B were unable to do
Table 4. Observed portfolio choices (across experiments)

<table>
<thead>
<tr>
<th>Type of behavior</th>
<th>RV</th>
<th>Strictly-RV</th>
<th>Ind</th>
<th>Non-RV</th>
<th>All</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\delta^A \geq \delta^B$</td>
<td>$\delta^A &gt; \delta^B$</td>
<td>$\delta^A = \delta^B$</td>
<td>$\delta^A &lt; \delta^B$</td>
<td></td>
</tr>
</tbody>
</table>

**Exp.1**

<table>
<thead>
<tr>
<th>Frequencies</th>
<th>80%</th>
<th>52%</th>
<th>29%</th>
<th>20%</th>
<th>100%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean $\delta^A$</td>
<td>0.626</td>
<td>0.715</td>
<td>0.465</td>
<td>0.307</td>
<td>0.563</td>
</tr>
<tr>
<td>Mean $\delta^B$</td>
<td>0.398</td>
<td>0.361</td>
<td>0.465</td>
<td>0.612</td>
<td>0.440</td>
</tr>
<tr>
<td>Nb. of obs.</td>
<td>73</td>
<td>47</td>
<td>26</td>
<td>18</td>
<td>91</td>
</tr>
</tbody>
</table>

**Exp.2**

<table>
<thead>
<tr>
<th>Frequencies</th>
<th>81%</th>
<th>45%</th>
<th>36%</th>
<th>19%</th>
<th>100%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean $\delta^A$</td>
<td>0.541</td>
<td>0.549</td>
<td>0.531</td>
<td>0.336</td>
<td>0.502</td>
</tr>
<tr>
<td>Mean $\delta^B$</td>
<td>0.365</td>
<td>0.231</td>
<td>0.531</td>
<td>0.606</td>
<td>0.411</td>
</tr>
<tr>
<td>Nb. of obs.</td>
<td>152</td>
<td>84</td>
<td>68</td>
<td>36</td>
<td>188</td>
</tr>
</tbody>
</table>

RV = risk-vulnerable; Ind = indifferent; A = without background risk; B = with background risk

**Result 2.** When background risk is either introduced or removed, there is a significantly large level of adjustment of the investment in the risky asset both for strictly-risk-vulnerable and for non-risk-vulnerable subjects.

**Support for result 2**

We compare the distributions of the absolute value of adjustments only for strictly-risk-vulnerable and non-risk-vulnerable subjects (we exclude indifferent subjects for which the adjustment equals zero). It is apparent from Table 2 that the mean adjustment is 0.330 for strictly-risk-vulnerable subjects and 0.282 for non-risk-vulnerable subjects, an insignificant difference (t-test, double-sided, $p = 0.187$; KS, $p = 0.787$). Furthermore, there is no significant difference in adjustment when background risk is introduced (treatment $AB$) or removed (treatment $BA$), neither for strictly-risk-vulnerable nor for non-risk-vulnerable subjects (KS test, two-sided, 5%).

It is important to observe that these adjustments are quite large compared it and therefore might have kept their investment at the same level as in situation $A$. On the other hand, the frequency of non-risk-vulnerable subjects is the same in both experiments (Fisher exact test, 5%).
to previous experimental findings. For instance, Lusk & Coble (2008, p. 334) found that “background risk has small to no effect on risk-taking behavior”. They observed a very slight increase in the mean number of safe choices (from 5.40 to 5.89) when subjects had to face a zero-mean background risk, while the median number of safe choices was unaffected (equal to 6 both with and without background risk). Similarly in the investment task analyzed by Lee (2008) the average absolute adjustment over ten rounds was 0.364 on a scale from 0 to 10. We believe that the key reason for such a strong difference between our results and the former ones, is that we relied on a within-subject design which allows to capture, separately, the impact of background risk on the level of investment of strictly-risk-vulnerable and non-risk-vulnerable subjects. The results summarized in Table 2 highlight the strong impact of the background risk on the portfolio choice for both types of non-indifferent subjects. When exposed to background risk, strictly-risk-vulnerable subjects divide roughly by two their level of investment (from 0.608 to 0.278) while non-risk-vulnerable subjects multiply roughly by two their investment (from 0.326 to 0.608). By contrast, if we had relied on a between-subjects design, with half of the subjects exposed to background risk (and the other half non-exposed), we would have found only a small difference of 0.101 between the average level of investment of the non-exposed (0.522) and the exposed (0.421) subjects, an approximate decrease of about 20% as shown by the last column of Table 1.

**Result 3.** The level of adjustment to background risk of the investment in the risky asset $\delta^A - \delta^B$ is neither affected by socio-demographic variables nor by treatment effects.

**Support for result 3**

In order to identify the variables that may affect the level of adjustment to background risk we rely on subjects’ socio-demographic data: gender (1 if women), age, number of siblings and religion (measured by the number of days of worship per month). Since the choice-space is not the same in *Exp.1* and *Exp.2*, we run a separate regression for each data set. Moreover, to provide a meaningful estimate for the difference $\delta^A - \delta^B$, we need to run censored regressions which capture the fact that subjects were constrained both for their choice of $\delta^A$ and for their choice of $\delta^B$. Consider for instance a subject who invests $\delta^A = 0.8$ without background risk. After background risk is introduced, his adjustment is constrained by his initial investment level, both downwards ($\delta^A - \delta^B \leq 0.8$) and upwards ($\delta^A - \delta^B \geq -0.2$). More generally, the following two inequalities hold: (i) $\delta^A - 1 \leq \delta^A - \delta^B \leq \delta^A - 0$ and (ii) $0 - \delta^B \leq \delta^A - \delta^B \leq 1 - \delta^B$.\footnote{To compute these boundaries, notice that: $\delta^A - \delta^B$ is minimum if $\delta^A = 0$ and $\delta^B = 1$ (left bound), and $\delta^A - \delta^B$ is maximum if $\delta^A = 1$ and $\delta^B = 0$ (right bound).} Based on these constraints, we define the censoring variable $cv$ as
follows:

\[ cv = \begin{cases} 
-1 & \text{if } \delta^A = 0 \text{ or } \delta^B = 1 \text{ or both (left-censored)} \\
0 & \text{if } 0 < \delta^A < 1 \text{ and } 0 < \delta^B < 1 \text{ (uncensored)} \\
1 & \text{if } \delta^A = 1 \text{ or } \delta^B = 0 \text{ or both (right-censored)} 
\end{cases} \]

The results of the censored regressions with robust standard errors are reported in Table 5.\textsuperscript{23} For \textit{Exp.1} neither the variable treatment (\textit{AB} or \textit{BA}) nor the socio-demographic variables affect the adjustment of the portfolio choice. However, we observe an insignificant gender effect in \textit{Exp.2} (\(p = 0.062\)) which is mainly due to a larger adjustment by males: on average \(\delta^A - \delta^B = 0.193\) (0.122) for male versus 0.065 (0.082) for women in \textit{Exp.2 (Exp.1)}. Because of the many differences between \textit{Exp.1} and \textit{Exp.2}, several reasons are likely to explain such effect, for instance the larger stakes or the windfall nature of endowments in \textit{Exp.2}. However, the likelihood of being risk-vulnerable is not affected by gender.\textsuperscript{24}

\textsuperscript{23} The regressions do not take into account the data of the 30 subjects for whom either \(\delta^A = \delta^B = 0\) or \(\delta^A = \delta^B = 1\), because for those subjects the censuring variable \(cv\) cannot be defined. Indeed, letting \(\tilde{\delta}^A\) and \(\tilde{\delta}^B\) stand for the unconstrained portfolios, subjects who expressed such choices could have chosen unconstrained portfolios such that, either \(\tilde{\delta}^A > \tilde{\delta}^B\), \(\tilde{\delta}^A < \tilde{\delta}^B\), or \(\tilde{\delta}^A = \tilde{\delta}^B\).

\textsuperscript{24} For each experiment (\textit{Exp.1} and \textit{Exp.2}), we ran a logit regression with dependent variable the sign of \(\delta^A - \delta^B\) (= 1 if \(\delta^A - \delta^B \geq 0\)). The explanatory variables were treatment, age, gender, religion and siblings. None of these variables affected the likelihood of being risk-vulnerable.
Table 5. Determinants of the level of adjustment to background risk ($\delta^A - \delta^B$)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Exp1</th>
<th>Exp2</th>
</tr>
</thead>
<tbody>
<tr>
<td>model</td>
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<td></td>
</tr>
<tr>
<td>treatment</td>
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<td>0.1316</td>
</tr>
<tr>
<td>age</td>
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</tr>
<tr>
<td>gender</td>
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<tr>
<td>religion</td>
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<td>-0.0015</td>
</tr>
<tr>
<td>_cons</td>
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<td>0.8465</td>
</tr>
<tr>
<td>sigma</td>
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<td></td>
</tr>
<tr>
<td>_cons</td>
<td>0.3886***</td>
<td>0.5009***</td>
</tr>
</tbody>
</table>

Statistics

| N   | 167 | 81  |
| N lc| 14  | 8   |
| N unc| 119 | 52  |
| N rc| 34  | 21  |

Legend: * p<.1; ** p<.05; *** p<.01

N=Nb. of obs.; lc=left-censored, unc=uncensored; rc=right-censored

6 Discussion and conclusions

Based on the results of two very different lab experiments we found that over 80% of our subjects are risk-vulnerable in the following sense: they do not increase their investment in the risky asset in favor of the safe asset when exposed to an actuarially neutral and independent additive background risk that affects their initial wealth. While our data adds to the already existing experimental evidence about risk vulnerability, it does so in a non-ambiguous way. First, in contrast to some previous experiments related to risk vulnerability, our experiment was designed on purpose for eliciting subjects’ risk vulnerability. Second, in contrast to previous experiments designed on purpose, our experiment relied on a within-subject comparison which seems more appropriate for the issue of risk vulnerability. Finally, in contrast to some previous experiments, our results are non-ambiguous and clear-cut. While the lab has the advantage to offer high control over the background risk that the experimenter can manipulate, stake sizes are necessarily limited and the background risk is bounded by the frame of the lab. It would be useful therefore to contrast our findings obtained with standard student subjects to data from field experiments for which the researcher would be able to compare individuals that
are exposed (versus unexposed) to some well-identified background risk (e.g. earthquakes, tsunamis, floods,...), or more generally individuals with variable and measurable exposure to some background risk.

References


